

# Bank Liability Structure

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## Abstract

We develop a dynamic continuous-time model of optimal bank liability structure that incorporates liquidity services on deposits, deposit insurance, capital requirement, regulatory closure, and endogenous default. Nesting the classic model of nonfinancial firms as a special case, the model shows how deposits allow an insured bank to maximize its value by combining with nondeposit debt and equity in its liability structure. The model shows that a value-maximizing bank balances between deposits and nondeposit debt so that the endogenous default coincides with the regulatory closure. Such balance maximizes the tax benefit of debt and minimizes the protection for deposits, leading to a leverage higher than the optimal leverage without deposits. Our comparative static analysis shows that the balance between deposits and nondeposit debt is very important in understanding banks' optimal responses to the changes in the business or in the regulation.

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# 1 Introduction

Bank liability structure has drawn much attention from regulators and the public after the crises experienced by the banking industry. Regulators around the world have gradually rolled out regulations on bank liability structure, and the shape of bank regulation is still evolving. After the frequent bank runs during the Great Depression, the Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC). After the financial crisis during the Great Recession, the Dodd-Frank Act of 2010 brought sweeping regulatory reforms ranging from FDIC deposit insurance to stress tests of banks' capital adequacy. Worldwide regulators agreed on Basel III in 2011 to strengthen restrictions on bank leverage.

While banks have been readjusting their capital structure to new regulations, both the academic and the regulators have been grappling with the questions about both the level and the composition of capital that banks should hold. There have been arguments for restricting bank leverage to a level similar to nonfinancial firms (see Admati and Hellwig, 2013). There are also antithetical views on whether banks should hold long-term bonds. Bulow and Klemperer (2013) suggest that banks hold just equity and securities that can be converted to equity but not nonconvertible debt. A Fed governor (Tarullo, 2013) goes in the opposite direction by arguing that holding more long-term bonds can improve the capital structure and the resolution of banks. The Dodd-Frank Act requires the FDIC to reform its deposit insurance policy, reducing or eliminating the deposit insurance subsidy, which is believed to give banks additional incentive for bank leverage. Some academics (e.g., Fleischer, 2013) even propose cutting the corporate tax rate to make banks safer because the tax benefits of interest expenses are a major reason for leverage.

The debate on bank capital regulation demonstrates the need for a better understanding of bank liability structure. Each regulation typically attempts to solve a problem observed in some parts of bank liability structures (see Santos, 2000). Arguments for a regulation often implicitly assume that other parts will remain unchanged, ignoring the response of banks that optimally adjust various parts of their liability structures. It is unclear whether the problem will be solved after banks' optimal responses to the regulation. It is even possible that the regulation may lead to unintended consequences. The optimal responses of banks potentially complicate the transformation from a regulation to its desired result.

Before working out banks' optimal responses to the proposed changes in regulations, we need to understand how a bank chooses leverage and liability structure when it maximizes its value. Value maximization is a fiduciary responsibility of bank management: acting in the interest of its claim-holders. Our focus on value maximization sets aside the principal-agent problems such as the management's conflict of interests with stake holders, although these problems may play roles in banks' choices of liability structures (Admati et al., 2013). Value-

maximizing banks do not maximize social welfare, such as reducing systemic risks or increasing banking services. An analysis of social welfare implications of bank leverage is unquestionably important, but understanding of the choices of liability structures by value-maximizing banks is necessary for a proper social welfare analysis of bank regulation.

Banks distinguish themselves from other firms by taking deposits. Deposits are different from other forms of debt partly because banks earn income from the provision of account services and liquidity services to depositors. In the literature, the account services or liquidity services associated with deposits are also called bank liquidity production. The income from these services is sometimes called the liquidity premium of deposits. We call it the account service income. Other important features of deposits are that deposits are often insured and that deposit-taking banks may be closed and liquidated by charter authorities if the bank capital drops below the required threshold, unless the bank can be recapitalized. The risk exposure of deposits covered by deposit insurance should be reflected in the insurance premium, unless the insurance is subsidized by the insurance provider.

We develop a dynamic structural model that incorporates the institutional features of the insured banks. Our model nests the structural model of nonfinancial firms pioneered by Merton (1974, 1977) and Leland (1994) as a special case. The consistency of our model of banks with the classic model of firms is important because it shows how the special institutional features related to deposits distinguish banks from other firms in liability structure decisions. With this model, we clearly show why the business of taking deposits pushes up banks' optimal leverage.

We analytically solve for the optimal liability structure of banks that issue nondeposit debt and common equity while taking deposits and providing account services. The solution to our model offers new perspectives on bank liability structures. We find it optimal for a value-maximizing bank to choose deposits and nondeposit debt so that its endogenous default coincides exactly with the regulatory closure. With this optimal choice of liability structure, the distance to default is the same as the distance to regulatory closure. This optimal structure of liabilities maximizes the tax benefits of debt and minimizes the protection for the deposits, leading to a leverage higher than the optimal leverage taking deposits.

The above property of the optimal liability structure has an intuitive economic reason. Because of the income from account services, deposits are cheaper than nondeposit debt as financing sources. A bank should generally prefer deposits to nondeposit debt when balancing the benefits of nondeposit debt against the potential bankruptcy loss. However, the nondeposit debt does not affect bankruptcy risk as long as the endogenous default does not happen before the regulatory closure. A bank should therefore issue as much nondeposit debt as possible for availing of the tax benefits but avoid making a default happen before the regulatory closure. Hence, the optimal nondeposit debt sets the endogenous default and the regulatory closure concurrent. Our comparative static analysis demonstrates that this optimal balance

between deposits and nondeposit debt is crucial in understanding banks' optimal responses to the changes in the business and regulatory environment.

Another new perspective offered by our model is the link between insurance premium and liability structure. On one hand, a bank's choice of leverage and liability structure affects the insurance premium that the bank has to pay. On the other hand, the insurance premium affects the bank's choice of leverage and liability structure. Our model incorporates this feedback channel, which is crucial in assessing the regulatory policies pertaining to bank liability structures. Our model shows that an insurance subsidy not only raises the optimal bank leverage but also affects the optimal combination of deposits and nondeposit debt.

Our model of bank liability structure has implications to bank capital requirements. Since the global financial crisis of 2007–2009, regulators have decided to raise capital requirement for banks. Basel III raises the capital requirement for all banks and imposes additional requirements for large banks that are regarded as systemically important. Both European and U.S. regulators have laid out new capital requirements for banks in accordance with Basel III. Some academics have proposed further raising the capital requirement. Two important questions for regulators are: how will banks adjust their liability structure in response to the increase in the equity ratio requirement? For example, it is important to know whether a value-maximizing bank cuts deposits or debt, or both, when lowering its leverage to meet the capital requirement. If we interpret the capital requirement as the minimum capital for a bank to operate without being closed, our model shows that banks respond to a tightening in the capital requirement by reducing deposits and increasing nondeposit debt in their liability structures.

Since banks use much higher leverage than nonfinancial firms do, corporate tax is particularly important for bank liability structures. Besides showing that the optimal bank leverage is lower in an economy with lower tax benefits, our model suggests that it is optimal for banks to shrink nondeposit debt more than deposits if the tax benefits are lowered. In addition, the model shows that banks remain substantially leveraged even when the tax benefits are nearly zero. While a full welfare analysis is needed for the tax policies, our model of the optimal responses of bank liability structures should lay a stepping stone for evaluating the benefits and costs of tax reforms in the context of bank leverage.

Our theory directly contributes to the literature of bank liability structure. Several papers have studied the reason for high leverage of banks. Without models or analysis, Buser et al. (1981) conceptually discuss banks that optimize FDIC-insured deposits in the presence of FDIC insurance. Song and Thakor (2007) examine banks' choice between uninsured deposits and nondeposit debt in a two-period model. DeAngelo and Stulz (2014) provide a rationale for leverage in a bank without deposit insurance, nondeposit debt, or asset risk. Garnall and Strebulaev (2013) posit that high leverage arises from low volatility of bank assets, while assuming a given mix of uninsured deposits and nondeposit debt. Allen and Carletti (2013) rationalize

high leverage of banks without nondeposit debt by assuming deposits as a cheaper funding source than equity. Our model complements the insights of these studies and formalizes the endogenous decision on both the leverage and the mix of the insured deposits and nondeposit debt for banks that face the risk of a regulatory closure and pays an endogenous premium for deposit insurance.

Our model extends the literature that attempts to apply the structural framework of Merton (1974, 1977) and Leland (1994) to bank capital structure. Leland considers unprotected debt and protected debt separately while interpreting the protected debt as rolling short-term debt. We analyze the endogenous choice of deposits and nondeposit debt simultaneously. The FDIC insurance and the regulatory closure make our model more complicated than Leland's model. Rochet (2008) applies the concept of endogenous default to the monitoring problem of banks while setting aside the optimal choice of leverage and liability structure. Harding et al. (2009) treat banks as the nonfinancial firms and regard all debt as deposits. Hugonnier and Morellet (2015) analyze uninsured banks in a modified version of our model. Our model incorporates the endogenous bank decision on the combination of insured deposits and nondeposit debt.

Our paper also extends the literature on the models of deposit insurance. In a setting with the institutional features, we derive the endogenous deposit insurance premium for an optimal liability structure. Several papers, notably Merton (1977) and Ronn and Verma (1986), have derived risk-adjusted deposit insurance policies. Duffie et al. (2003) price deposit insurance for a given leverage and given bankruptcy risk. Our work extends their models to incorporate the endogenous decisions on both the default and closure boundaries when a bank liability structure optimally adjusts for the endogenous insurance premium charged by the FDIC.

The road map for the rest of the paper is as follows. Section 2 develops the model of bank liability structure. Section 3 characterizes the optimal liability structure. Section 4 analyzes the comparative statics, showing how a bank liability structure optimally responds to the changes in exogenous factors. Section 5 concludes and discusses potential applications and extensions.

## **2 Bank Liability Structure**

While banks share some common characteristics with nonfinancial firms, banks differ from nonfinancial firms in that they take deposits and provide liquidity services to their depositors through check writing, ATMs, and other transaction services such as wire transfers, bill payments, etc. The banking business of taking deposits and serving accounts is heavily regulated in most countries. In the U.S., a large part of deposits are insured by the FDIC, which charges insurance premium and imposes regulations on banks. The model of FDIC insurance has gained popularity outside the U.S., and an increasing number of countries have started to offer deposit insurance. The International Association of Deposit Insurers (IADI) was formed

on May 6, 2002 to enhance the effectiveness of deposit insurance systems by promoting guidance and international cooperation. At the end of 2014, IADI represents seventy-nine deposit insurers from seventy-six countries and areas. Governments also impose regulations on both the opening of new banks and the closing of existing banks.

Deposits and the associated services, deposit insurance, and the regulations on opening and closing banks distinguish banking business from other nonfinancial corporate business and make the financial decisions of banks different from those of the other firms. Both nonfinancial firms and banks have access to cash flows generated by their assets and both finance their assets by issuing debt and equity. Firms operate in a market with two frictions: corporate taxes and bankruptcy costs. These frictions are crucial for firms in their choice of liability structure, as recognized in the literature originating from Modigliani and Miller (1963) and Baxter (1967) and analyzed in structural models by Leland (1994). Banks face these frictions too, but they have to incorporate simultaneously other considerations, such as deposit insurance, minimum capital requirement, and regulatory closure, in determining their optimal leverage and liability structure. Figure 1 illustrates the liability structure of a typical bank. In Section 2.1, we discuss each part of the structure in detail.

| <b>Asset Side</b>   | <b>Liability Side</b>  |
|---|--|
| Asset value: $V$<br>(volatility: $\sigma$ )<br>(cash flow: $\delta$ ) | Deposits: $D$<br>(benefit: tax deduction $\tau$ )<br>(benefit: service income $\eta$ )<br>(cost: bankruptcy $\alpha$ )<br>(cost: insurance premium $I$ ) |
|   | Nondeposit debt: $B$<br>(benefit: tax deduction $\tau$ )<br>(cost: bankruptcy $\alpha$ )   |
|   | Equity: $E$<br>(tangible equity: $T = V - D - B$ )<br>(charter value: $E - T$ )  |
| Charter value: $F - V$  |  |
| Bank value: $F = D + B + E$   |  |

Figure 1: An Illustration of Bank Liability Structure

## 2.1 Assets and Liabilities

A typical bank owns a portfolio of risky assets that generate cash flows. The portfolio of assets is valued at  $V$ , which is a major part of Figure 1. The asset portfolio is risky, and its value follows a stochastic process. Following Merton (1974) and Leland (1994), we assume that the stochastic process is a geometric Brownian motion

$$dV = (r - \delta)Vdt + \sigma VdW, \quad (1)$$

where  $r$  is the risk-free interest rate,  $\delta$  is the rate of cash flow,  $\sigma$  is the volatility of asset value, and  $W$  is a Wiener process in the risk-neutral probability measure. The instantaneous cash flow of the assets is  $\delta V$ , which is paid as either dividend to equity holders or liabilities to the other stakeholders. In a nonfinancial firm,  $\delta V$  is the total earnings before interests, but in a bank,  $\delta V$  represents only the earnings from bank assets such as loans, not including the income from serving deposit accounts. The risk of the asset portfolio is described by the volatility  $\sigma$  of the asset value. Notice that the cash flow  $\delta V$  follows a geometric Brownian motion with the same volatility  $\sigma$ . One may start with the assumption that the asset cash flow follows a geometric Brownian motion with volatility  $\sigma$  and then show that the asset value follows the stochastic process in equation (1).

We focus on the liability structure for a given portfolio of assets. This focus rules out interesting issues of endogenous asset substitution. The literature has pointed out that corporate debt may create incentives to substitute assets with higher risk (e.g., Green, 1984, and Harris and Raviv, 1991) and FDIC insurance may also make for such an incentive (e.g., Pennacchi, 2006, and Schneider and Tornell, 2004). Although the endogenous choice of assets along with the choice of liability is an interesting research topic, our study can be viewed as an analysis of the optimal liability structure of a bank that has already optimally chosen its asset portfolio.

Following Merton (1974) and Leland (1994), we assume that investors have full information about the asset value. In reality, active investors use all available information to assess bank asset value and cash-flows although only accounting values of assets are directly observable in quarterly filing. The full-information assumption sets aside the disparity between accounting value and intrinsic value. We interpret  $V$  as the fair accounting value. If the assets are of the same risk category, we may interpret  $V$  as the value of risk-weighted assets. Also following Leland (1994), we assume  $V$  is the after-tax value of assets, and thus  $\delta V$  is the after-tax cash flow. Alternatively, one may specify the before-tax value of the assets as in Goldstein et al. (2001).

Banks take deposits from households and businesses and provide account services to depositors. Deposits, the first part on the liability side in Figure 1, are an important source of funds for banks to finance their assets. Let  $D$  denote the deposits that a bank takes. Deposits are rendered safe if banks purchase insurance that guarantees depositors in full. We assume all deposits are insured to keep our model simple and focus on the tradeoff between the insured deposits and other debt. The insurance requires a bank to pay a premium, which will be discussed in the next subsection. If deposits are risk-free, the fair interest rate on deposits is the risk-free rate. However, banks typically pay lower interest rates on deposits. Banks also charge fees for services such as money transfers, overdrafts, etc. Depositors accept a lower or zero interest rate because they receive the services associated with maintaining their accounts and transacting normal payments. The Banking Act of 1933, known as the Glass-Steagall Act,



prohibited banks from paying interest on demand deposits and gave the Fed the authority to impose ceilings on interest rates paid on time deposits. The prohibition and ceiling of interest rate on deposits were removed after the Depository Institutions Deregulation and Monetary Control Act of 1980 and the Depository Institutions Act of 1983, the latter of which is known as the Garn-St. Germain Act.

Let  $r$  be the risk-free interest rate,  $\eta_1$  be the discount for liquidity services of deposits, and  $\eta_2$  be the banks' fee income on each dollar of deposits. A bank's net liability on deposits is  $C_D = (r - \eta_1)D - \eta_2D$ , excluding the deposit insurance premium. Let  $\eta = \eta_1 + \eta_2$ , which is the net income on each dollar of deposits. The net deposit liability is  $C_D = (r - \eta)D$ , excluding deposit insurance premium. The bank's total liability on deposits is  $I + C_D$ , where  $I$  is the insurance premium. The parameter  $\eta$  plays a crucial role in our model of banks. It represents a sacrifice in the required rate of return that the households are willing to accept for the services provided by the bank. This sacrifice is a distinctive character of deposits. The low-cost of deposits can also be attributed to some local monopoly rents that the banks enjoy because of the barrier of entry into the banking industry. We do not explicitly analyze the market for deposits but instead focus on the effect of deposits on the bank's choice of liability structure.

The other important sources of banks' funding are nondeposit debt, which is the second part on the liability side in Figure 1. There are no account services associated with the nondeposit debt. The debt pays interest until bankruptcy, at which it has a lower priority than the deposits in claiming the liquidation value of the bank assets. The lower priority potentially makes the nondeposit debt protect the deposits at bankruptcy. For this reason, some unsecured debt qualifies for being Tier 2 capitals in bank regulations. The nondeposit debt comes with costs: their yield contains a credit premium to compensate the debt holders for bearing the risk of bankruptcy. The credit premium arises endogenously in our model; it depends on both the risk of assets and the liability structure. Thus, a bank's choice of liability structure affects the credit premium, which we solve endogenously along with the debt value. Let the liability on nondeposit debt be  $C_B$  and the debt value be  $B$ . The pricing equation of nondeposit debt is

$$\frac{1}{2}\sigma^2V^2B'' + (r - \delta)VB' - rB + C_B = 0, \quad (2)$$

where  $B'$  and  $B''$  are the first and second derivatives of  $B$  with respect to  $V$ .

The common equity holders garner all the residual value and earnings of the bank after paying the contractual obligations on the deposits and the nondeposit debt. The first slice of value that equity holders lay claims to is the asset value exceeding the deposits and the nondeposit debt:  $T = V - (D + B)$ . This slice, also shown on the liability side in Figure 1, is called tangible equity or book-value of equity. This is the value that equity holders would receive if the bank assets are liquidated at the fair value and if all the deposits and the nondeposit debt is paid off at its par value. A larger tangible equity means a smaller loss for depositors and

debt holders after liquidation. Hence, regulators regard the tangible equity as a bank capital of the highest quality—the core Tier 1 capital.

Since equity holders receive all future earnings of the bank, the present value of the future earnings is the bank’s charter value, which is the bottom part of the equity in Figure 1. The earnings contain the savings from corporate tax. Since interest expenses are deductible from earnings for tax purposes, the flow of tax savings is  $\tau(I + C_D + C_B)$ . The dividend paid to the equity holders is the difference between the asset cash flow and the after-tax liability associated with the deposits and the nondeposit debt:  $\delta V - (1 - \tau)(I + C_D + C_B)$ . The pricing equation of equity value  $E$  before bankruptcy is

$$\frac{1}{2}\sigma^2V^2E'' + (r - \delta)VE' - rE + \delta V - (1 - \tau)(I + C_D + C_B) = 0, \quad (3)$$

where  $E'$  and  $E''$  are the first and second derivatives of  $E$  respect to  $V$ . Since the equity value depends on its dividend, it is affected by the liability structure. The triplet  $(I, C_D, C_B)$  therefore characterizes the liability structure of a bank.

## 2.2 Deposit Insurance and Regulations

Without deposit insurance, borrowing through deposits brings the risk that depositors may run, a major challenge commonly faced by banks but not by nonfinancial firms. As experienced in the crises of the U.S. banking history and theorized in the academic literature, depositors may run from a bank if they believe it has difficulty in repaying their deposits promptly upon their demand. When depositors run, the bank will be closed, unless it is recapitalized to stop the run, and its assets will be liquidated. The bank run literature was pioneered by Diamond and Dybvig (1983), who construct a model in which bank run emerges as an equilibrium. The literature has been extended significantly by Allen and Gale (1998) and others. The establishment of the FDIC is to deter bank runs by insuring that the deposits are paid when a bank closes. The FDIC deposit insurance limit was raised from \$100,000 to \$250,000 on October 3, 2008 to prevent bank runs during the global financial crisis. In our model of banks, all deposits are insured. Our model does not address the risk of bank run. A bank in our model is closed either by its owners or by its regulator.

The equity holders of a bank can choose to default before the regulatory closure. Absent regulatory closure, there is an optimal point for the equity holders to default. The default decision maximizes equity value. The optimal default is called the *endogenous default* and derived by Leland (1994) for firms without deposits. In section 3.1, we provide the formula of endogenous default in the presence of both deposits and nondeposit debt. Let  $V_d$  be the point of endogenous default. Without regulatory closure, the equity holders choose to default if and only if the asset value  $V$  reaches  $V_d$ .

A bank may be closed by its charter authority, which is typically either the bank's state banking commission or the Office of the Comptroller of the Currency (OCC). The charter authority closes a bank if the bank is insolvent or if the bank's capital is deemed to be too low to be sustainable. For example, a bank is categorized by regulators as critically under-capitalized when the total capital that protects deposits drops to a threshold. In principle, the FDIC categorizes a bank as critically under capitalized when the total capital drops to 2% of its asset value. The FDIC closes the bank if the bank cannot be recapitalized. For a review of the rules for the list of critically under-capitalized banks, we refer readers to Shibut et al. (2003). The total capital is the sum of Tier 1 and Tier 2 capital. In our model, it is the sum of the tangible equity and the nondeposit debt. This is equivalent to  $[V - (D + B)] + B = V - D$ . Let  $V_a$  be the threshold when the charter authority closes the bank. If the threshold is a fraction  $\beta$  of its asset value (say,  $\beta = 2\%$ ), then  $V_a - D = \beta V_a$ , which implies  $V_a = D/(1 - \beta)$ .

The closure rule in our model may also be interpreted as the capital requirement, the minimum capital for a bank to operate, as modeled in Rochet (2008). Under the capital requirement, the charter authority shuts down the bank, when the total capital falls below the capital requirement. A capital requirement is typically specified as a ratio to the asset value. If the capital requirement is ten percent, then  $\beta = 10\%$ .

The FDIC functions as a receiver of the closed banks and as an insurer of the banks' deposits. As a receiver, the FDIC liquidates the assets of a closed bank in its best effort to pay back the bank's creditors. Suppose the liquidation cost is  $\alpha V_a$ , proportional to the asset value  $V_a$  when the bank is closed. The cost of liquidation by the FDIC may be different from the costs of liquidation through bankruptcy courts. Since the FDIC does not go through the lengthy procedure of bankruptcy, it is likely that the FDIC liquidation cost is smaller than the typical bankruptcy cost in the private sector. Title II of the Dodd-Frank Act reflects the belief that the cost of FDIC liquidation is lower than the cost of bankruptcy procedures. Title II authorizes the FDIC to receive and liquidate the failed large financial institutions to avoid lengthy and costly bankruptcy procedures, which are supposed to be harmful for the stability of financial system.

As an insurer, the FDIC pays  $D$  to the depositors when the bank is closed. The insurance corporation loses  $D - (1 - \alpha)V_a$  if  $(1 - \alpha)V_a < D$ , or otherwise it loses nothing. Thus, the loss function is  $[D - (1 - \alpha)V_a]^+$ , where  $[x]^+ = x$  if  $x \geq 0$  and  $[x]^+ = 0$  if  $x < 0$ . Since  $V_a = D/(1 - \beta)$ , the loss function is positive if  $\beta < \alpha$ , in which case the FDIC expects to suffer a loss after a bank closure. In practice, the FDIC always expects a chance of loss because the liquidation cost is uncertain. To keep our analysis tractable, we assume a fixed  $\alpha$  and assume  $\beta < \alpha$  so that the FDIC expects a loss at bankruptcy.

Therefore, a bankruptcy happens if the liabilities of the bank are defaulted by the equity holders endogenously or if the bank is closed by the regulators when its capital falls below the minimum requirement. Mathematically, the point of bankruptcy is  $V_b = \max\{V_a, V_d\}$ . When

the bank assets are liquidated after a bankruptcy, the depositors are paid first, and the debt holders are paid the next if there is value left. Since  $\alpha V_b$  is the bankruptcy cost, the payoff to the debt holders is  $[(1 - \alpha)V_b - D]^+$ . Therefore, there is a boundary condition for the debt value: the debt holders receive  $B(V_b) = [(1 - \alpha)V_b - D]^+$  at bankruptcy. There is also a boundary condition for the equity value:  $E(V_b) = 0$  when the bankruptcy wipes out equity.

To cover its potential loss, the FDIC charges insurance premiums. In 2006, Congress passed reforms that permit the FDIC to charge risk-based premiums. For deposit insurance assessment purposes, an insured depository institution is placed into one of four risk categories each quarter, depending primarily on the institution's capital level and supervisory evaluation. A riskier bank pays a higher insurance premium than a safer bank does. Recall that  $I$  denotes the deposit insurance premium that a bank pays. Until 2010, the FDIC assesses insurance premiums based on total deposits. The assessment rate is  $a$  such that  $I = aD$ . There have long been concerns that banks shift deposits out of balance sheets temporally at quarter-ends to lower the assessment base. Since April 2011, the FDIC has changed the assessment base to be the difference between the risk-weighted assets and the tangible equity, as required by the Dodd-Frank Act (Section 331). In our model, the new assessment base equals  $D + B$ , which implies that assessment rate is  $b$  such that  $I = b(D + B)$ . The actual premium assessment may also depend on the credit rating and how the deposits are protected by the nondeposit debt. For more details, see Federal Deposit Insurance Corporation (2011).

### 3 Valuation and Optimization

Table 1 summarizes the exogenous parameters in the model and the assumptions on them. In the table, service income is positive but with a rate smaller than the risk-free rate:  $0 < \eta < r$ . Corporate tax is present:  $0 < \tau < 1$ . The liquidation by the FDIC or at default is costly:  $0 < \alpha < 1$ . These assumptions are not only realistic but also the requisite mathematical conditions for valuation and optimization.

#### 3.1 Bank Value and Insurance Premium

The debt value and equity value are affected by the risk of bankruptcy. The Arrow-Debreu state price of bankruptcy plays a key role in our bank valuation. Consider a security that pays \$1 when the bankruptcy occurs, and otherwise it pays nothing. The price of this security is the state price of bankruptcy. According to Merton (1974), the state price is  $[V_b/V]^\lambda$ , where  $\lambda$  is the positive root of the following quadratic equation:

$$\frac{1}{2}\sigma^2\lambda(1 + \lambda) - (r - \delta)\lambda - r = 0. \quad (4)$$

| Parameter               | Notation | Allowed range |
|-------------------------|----------|---------------|
| Asset volatility        | $\sigma$ | $(0, \infty)$ |
| Asset cash flow         | $\delta$ | $[0, \infty)$ |
| Asset value             | $V$      | $(0, \infty)$ |
| Risk-free interest rate | $r$      | $(0, \infty)$ |
| Bank service income     | $\eta$   | $(0, r)$      |
| Corporate tax benefit   | $\tau$   | $(0, 1)$      |
| Liquidation cost        | $\alpha$ | $(0, 1)$      |
| Capital requirement     | $\beta$  | $(0, \alpha]$ |
| Insurance subsidy       | $\omega$ | $[0, 1)$      |

Table 1: Exogenous parameters are pre-specified, not determined by either valuation or optimization in the model. The allowed ranges are assumptions of the model.

The quadratic equation implies that  $\lambda$  is an increasing function of  $r$  and a decreasing function of  $\delta$  and  $\sigma$ . If the cash flow of the assets is zero, i.e.,  $\delta = 0$ , we have  $\lambda = 2r/\sigma^2$ , which is proportional to  $r$  and inversely proportional to  $\sigma^2$ .

Bank value depends on its liability structure  $(I, C_D, C_B)$  because the liabilities affect the bankruptcy boundary and its state price. The following theorem, derived in Appendix A.1, summarizes the relation between bank value and liability structure.

**Theorem 1** *Given a liability structure  $(I, C_D, C_B)$ , the bankruptcy boundary is*

$$V_b = \max \left\{ \frac{C_D}{(r - \eta)(1 - \beta)}, (1 - \tau) \frac{\lambda}{1 + \lambda} \cdot \frac{I + C_D + C_B}{r} \right\}. \quad (5)$$

*The total deposits, the debt value, the equity value, and the bank values are, respectively,*

$$D = \frac{C_D}{r - \eta}, \quad (6)$$

$$B = \frac{C_B}{r} \left( 1 - \left[ \frac{V_b}{V} \right]^\lambda \right) + \left[ (1 - \alpha)V_b - \frac{C_D}{r - \eta} \right]^+ \left[ \frac{V_b}{V} \right]^\lambda, \quad (7)$$

$$E = V - (1 - \tau) \frac{I + C_D + C_B}{r} \left( 1 - \left[ \frac{V_b}{V} \right]^\lambda \right) - V_b \left[ \frac{V_b}{V} \right]^\lambda, \quad (8)$$

$$F = V + \left( \frac{\eta C_D}{r(r - \eta)} + \frac{\tau(C_D + C_B) - (1 - \tau)I}{r} \right) \left( 1 - \left[ \frac{V_b}{V} \right]^\lambda \right) - \min \left\{ \alpha V_b, V_b - \frac{C_D}{r - \eta} \right\} \left[ \frac{V_b}{V} \right]^\lambda. \quad (9)$$

In equation (7), the value of the nondeposit debt is the expected value of the interests on the debt before the bankruptcy plus the expected recovery value at bankruptcy. In equation (8), the equity value is the residual asset value after subtracting the expected after-tax liabilities on the insurance, the deposits, and the nondeposit debt and after deducting the expected value of bankruptcy loss. In equation (9), which is for the bank value, the first term is the asset value,

the next term reflects the value of service income and the value of tax savings after subtracting the deposit insurance premium, and the last term reflects the value of bankruptcy loss.

Theorem 1 shows the role of service income and deposit insurance in bank valuation. Along with the tax benefits of interest expenses, account service income ( $\eta$ ) increases the bank value, as shown by the last term on the right-hand side of equation (9). The ability of the bank to attract deposits at a rate lower than the risk-free rate comes at a price: the bank incurs bankruptcy cost when the regulator closes the bank because the tangible equity drops to the minimum capital requirement. Moreover, the insurance premium reduces the bank value, which is evident in the last term of the equation.

We obtain the endogenous credit premium of the nondeposit debt from Theorem 1. The endogenous credit premium is  $C_B/B - r$ , where  $B$  depends on  $C_B$  as given by equation (7). The credit premium takes the probability of bankruptcy into account through the state price  $P_b$ ; simultaneously, the state price is affected by the liability structure. The insurance premium affects the credit premium although  $I$  does not appear in equation (7) explicitly. The premium affects the bankruptcy boundary  $V_b$  in equation (5), which in turn affects its state price. Then, the bankruptcy boundary and its state price affect the credit premium directly.

Our model shows the difference between banks and nonfinancial firms. If we set  $C_D = I = 0$  but  $C_B > 0$ , the formulas in Theorem 1 reduce to those in Leland (1994) for nonfinancial firms that issue only equity and unprotected debt. Leland's capital structure theory about nonfinancial firms is not applicable to banks that take deposits, earn service income, pay deposit insurance premium, and face the capital requirement and the risk of bank closure. Our model extends Leland's model to banks and offers a consistent framework for understanding the similarities and differences between banks and nonfinancial firms.

While the deposit insurance premium is exogenously given in Theorem 1, it should endogenously depend on the deposits under insurance and the risk involved in the bank. In principle, the insurance corporation should charge the bank a fair insurance premium. A fair premium makes the insurance contract worth zero to each party of the insurance contract. The next theorem, derived in Appendix A.2, characterizes the fair insurance premium.

**Theorem 2** *Given  $D$  dollars of deposits, the fair insurance premium is*

$$I^\circ = \frac{r[\alpha - \beta]^+}{1 - \beta} \cdot \left[ \frac{D}{(1 - \beta)V} \right]^\lambda \left( 1 - \left[ \frac{D}{(1 - \beta)V} \right]^\lambda \right)^{-1} D. \quad (10)$$

The above formula essentially setting the expected present value of the insurance premium paid to the insurance corporation equal to the expected present value of the insurance obligations at the bank closure. If  $\beta < \alpha$ , the fair premium  $I^\circ$  is positive. It converges to zero as  $\beta$  rises to  $\alpha$ . If  $\beta \geq \alpha$ , the fair premium is zero because the bank will be closed with enough asset value to cover the deposits in full.

The fair insurance premium  $I^\circ$  increases with  $D$ . If the deposits increase, not only the insurance premium increases, the assessment rate of the insurance premium also increases. The assessment rate is the premium for insuring one dollar deposits. That is, the assessment rate is  $I^\circ/D$ . The rate is increasing with  $D$  because an expansion of deposits exposes the insurance corporation to a bigger risk.

Duffie et al. (2003) argue that the FDIC does not charge enough insurance premium to cover its risk exposure. They also suggest that a lower premium may be necessary to compensate the insured banks for the costs of reporting requirements and tight regulation. A premium lower than the fair rate provides subsidized insurance to banks. To allow a subsidized insurance premium, we assume that the insurance premium is  $I = (1 - \omega)I^\circ$ , where  $\omega = 0$  represents a fair premium and  $\omega > 0$  represents a subsidized premium. With the subsidy, the assessment rate of the insurance premium is  $I/D$ , which is insurance premium per dollar of deposits. If the insurance corporation subsidizes deposit insurance, it increases the bank value because the bank does not pay enough premium for enjoying the risk-free value of deposits.

Even with a subsidy, the assessment rate, as well as the total premium that the bank pays, still endogenously depends on the deposits and the bank's risk profile. A liability structure with an endogenous insurance premium is characterized by the pair  $(C_C, C_B)$  because  $C_D$  determines  $D$ , which in turn determines both  $I^\circ$  and  $I$ . To be explicit, we can use equation (6) to relate the endogenous insurance premium directly to the deposit liability:

$$I = \frac{r(1 - \omega)[\alpha - \beta]^+ C_D}{(r - \eta)(1 - \beta)} \cdot \left[ \frac{C_D}{(r - \eta)(1 - \beta)V} \right]^\lambda \left( 1 - \left[ \frac{C_D}{(r - \eta)(1 - \beta)V} \right]^\lambda \right)^{-1}. \quad (11)$$

### 3.2 Optimal Liability Structure

We now examine how a value-maximizing bank chooses its liability structure. As pointed out earlier, the liability structure of a bank with an endogenous insurance premium is described by the pair  $(C_D, C_B)$ . An optimal liability structure is a deposit liability  $C_D^*$  and a nondeposit debt liability  $C_B^*$  that maximize the bank value in equation (9) subject to equation (11). The value-maximizing bank in our framework is fully aware that any decision pertaining to the liability structure has a consequence on the insurance premium. The bank should therefore be mindful of this channel in its choice of liability structure. The endogenous relation between the insurance premium and the liability structure captures the feedback channel from the insurance corporation to the bank and vice versa.

The next theorem, derived in Appendix A.3, provides a characterization of the optimal liability structure for a bank that pays an endogenous insurance premium and faces the risk of both endogenous default and regulatory closure.

**Theorem 3** *Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $0 < \beta \leq \alpha < 1$ , and  $0 \leq \omega < 1$ . Suppose the*

insurance premium is determined by equation (11). A liability structure with  $V_d < V_a$  is never optimal for an insured bank. There exists  $\epsilon \in [0, \alpha]$  such that for all  $\beta \in (\epsilon, \alpha]$ , the optimal structure is unique and satisfies  $V_d^* = V_a^*$ . In the optimal structure, the state price of bankruptcy is

$$\pi = \frac{1}{1 + \lambda} \cdot \frac{\eta(1 - \tau)\lambda(1 - \beta) + r\tau(1 + \lambda)}{\eta(1 - \tau)\lambda(1 - \beta) + r\tau(1 + \lambda) + r(1 - \tau)\lambda[(1 - \omega)\alpha + \omega\beta]}. \quad (12)$$

The optimal deposit liability and debt liability as ratios to assets are, respectively,

$$\frac{C_D^*}{V} = (r - \eta)(1 - \beta)\pi^{1/\lambda}, \quad (13)$$

$$\frac{C_B^*}{V} = \left( \frac{\tau}{1 - \tau} + \beta + (1 - \beta) \frac{\eta}{r} \frac{\lambda}{1 + \lambda} \right) \frac{r\pi^{1/\lambda}}{1 - \pi} + \frac{r\pi^{1/\lambda}}{\lambda}. \quad (14)$$

The theorem characterizes the optimal liability structure of a bank that enjoys tax benefit on interest expenses, takes deposits to earn service income, pays an endogenously determined insurance premium, and bears the risk of costly bankruptcy if the total capital falls below the minimum requirement. The formula in equation (12) shows that the optimal state price depends on the following exogenous parameters:  $r$ ,  $\sigma$ ,  $\delta$ ,  $\eta$ ,  $\alpha$ ,  $\beta$ ,  $\omega$ , and  $\tau$ . The parameters  $\sigma$ ,  $\delta$ ,  $\eta$  and  $\alpha$  are likely to be heterogenous among banks, whereas the parameters  $\beta$ ,  $\omega$ , and  $\tau$  are characteristics of the regulations and tax policy.

The theorem states that the optimal nondeposit debt makes the endogenous default boundary coincide with the bank closure boundary. Deposits attract a discount in the deposit rate as well as service fees, besides savings from the corporate tax. The cost of taking deposits is the insurance premium and the expected bankruptcy loss. By contrast, the nondeposit debt brings savings from the corporate tax but produces no account services or fee income; its cost is also an expected loss in bankruptcy. Therefore, at the margin, the bank should first use deposits to balance the benefits of leverage with the expected bankruptcy loss. With this balance, the bank should take as much nondeposit debt as possible to avail the tax benefits but should avoid the expected bankruptcy loss caused by the endogenous default. Therefore, the bank should not set the endogenous default boundary above the regulatory closure boundary. Hence, the optimal nondeposit debt should make the endogenous default occur at exactly the same point at which the regulatory closure happens.

If the asset volatility  $\sigma$  and the liquidation cost  $\alpha$  are both very high, it is possible for a liability structure with  $V_d > V_a$  to be optimal for some  $\beta$  close to 0. We have confirmed this possibility by both mathematical derivation and numerical optimization. If  $\sigma$  and  $\alpha$  are very high and  $\beta$  is very low, the endogenous insurance assessment rate may be so high that makes deposits too expensive compared to the other types of debt. When that happens, reducing deposits to have  $V_a < V_d$  may be optimal. Preventing the insurance assessment rate from being too high is the reason for  $\beta$  to be higher than a threshold  $\epsilon$  in Theorem 3. Nevertheless, for all the asset volatility and the liquidation cost we consider in section 4, we find  $\epsilon = 0$ . In this



case,  $V_a^* = V_d^*$  and the formulas in Theorem 3 hold for all  $\beta \in (0, \alpha]$ .

Theorem 3 incorporates the endogenous insurance premium in the optimal liability structure. The bank takes the cost of deposit insurance into account when considering the tradeoff among the tax benefit, the account service income, the minimum capital requirement, and the bankruptcy loss at regulatory closure. Since the insurance assessment rate is an increasing function of the deposit liability  $C_D$ , the bank considers both the increase in the premium caused directly by the expansion of deposits as well as the increase in the premium caused indirectly through the rise of the assessment rate.

Combining Theorem 3 with Theorems 1 and 2, we obtain the financial ratios, the insurance premium, and the credit premium in the optimal capital structure.

**Theorem 4** *In the optimal liability structure described by Theorem 3, the bankruptcy boundary relative to the asset value is  $V_b^*/V = \pi^{1/\lambda}$ , where  $\pi$  is given in equation (12). In addition, we have*

$$\frac{D^*}{V} = (1 - \beta)\pi^{1/\lambda} \quad (15)$$

$$\frac{B^*}{V} = \left( \frac{\tau}{1 - \tau} + \beta + (1 - \beta)\frac{\eta}{r} \frac{\lambda}{1 + \lambda} + \frac{1 - \pi}{\lambda} \right) \pi^{1/\lambda} \quad (16)$$

$$\frac{E^*}{V} = 1 - \left( 1 + \frac{1 - \pi}{\lambda} \right) \pi^{1/\lambda} \quad (17)$$

$$\frac{F^*}{V} - 1 = \left( \frac{\tau}{1 - \tau} + (1 - \beta)\frac{\eta}{r} \frac{\lambda}{1 + \lambda} \right) \pi^{1/\lambda} \quad (18)$$

$$\frac{I^*}{D^*} = r(1 - \omega) \frac{\alpha - \beta}{1 - \beta} \cdot \frac{\pi}{1 - \pi} \quad (19)$$

$$\frac{C_B^*}{B^*} - r = \frac{r\pi}{1 - \pi}. \quad (20)$$

In this theorem, the optimal liability structure is characterized by a set of financial ratios endogenously determined by bank management that maximizes the bank value. Although deposits are partially determined by the supply in the markets, banks have control of the ratios because they can adjust the assets to achieve their desired capital structure. The ratio of deposits to assets,  $D^*/V$ , is an endogenous variable of our interest. The ratio of the nondeposit debt to assets,  $B^*/V$ , is another important endogenous variable. The optimal leverage of the bank is measured by the ratio of the tangible equity to assets  $T^*/V$ , where  $T^* = V - D^* - B^*$ . A higher leverage corresponds to a lower tangible equity ratio. It is useful to point out that the tangible equity of a bank can be negative in practice. For example, the U.S. operation of Deutsche Bank reported a total asset value of \$355 billion and a negative \$5.68 billion Tier 1 capital in its December 2011 filing as a bank holding company.

The deposits and the nondeposit debt together determine several other endogenous variables. They determine the boundary of bank closure,  $V_a^*/V$ , and the boundary of endogenous default,  $V_d^*/V$ , relative to the asset value. The two boundaries determine the distance to

bankruptcy. The higher of the two is the bankruptcy boundary  $V_b^*/V$ . In the optimal capital structure described in Theorem 4, we have  $V_a^*/V = V_d^*/V = V_b^*/V = \pi^{1/\lambda}$ , where  $\pi$  is the state price of bankruptcy. The possibility of the bankruptcy causes the bank to pay a credit premium,  $C_B^*/B^*$ , on the nondeposit debt. The credit premium is often observable in the market. The insurance premium per dollar of deposits,  $I^*/D^*$ , is also endogenously determined by the optimal liability structure. The account service income and the tax benefit create the charter value of the bank, which is the difference between the bank value and the asset value. The charter value is reflected in the ratio of the bank value to its assets,  $F^*/V$ , or it can be measured by  $F^*/V - 1$ .

## 4 Comparative Statics

The optimal bank liability structure depends on the characteristics of bank business. The most important characteristics are the account service income  $\eta$ , the asset volatility  $\sigma$ , the bankruptcy cost  $\alpha$ , the minimum capital requirement  $\beta$ , the insurance subsidy  $\omega$ , and the corporate tax rate  $\tau$ . We examine the effects of these characteristics on the optimal bank liability structure in comparative static analyses. Before analyzing the comparative statics, we first examine the effects of deposits on the optimal bank liability structure.

### 4.1 Effects of Deposits

To examine the effects of deposits, we imagine a bank that does not take deposits. Such a bank is not affected by the parameters  $\eta$ ,  $\beta$ , or  $\omega$ . These three parameters are special for banks that take deposits and are regulated by the charter authorities and insured by the FDIC. These parameters distinguish the optimal liability structure in Theorems 3 and 4 from the optimal liability structure of firms analyzed by Leland (1994). A firm in Leland's model issues nondeposit debt and equity but does not take deposits. If a bank in our model does not take deposits, its liability structure is equivalent to a firm in Leland's model.

For convenience of comparison, we present the optimal structure of a firm in Leland's model in terms of our notations. Let  $\bar{V}_b^*$  be the bankruptcy boundary,  $\bar{B}^*$  be the debt value,  $\bar{E}^*$  be the equity value, and  $\bar{F}^*$  be the firm value in the optimal liability structure of the firm. Let  $C_{\bar{B}}^*$  be the coupon rate of the debt in the optimal structure. The credit premium in the optimal structure is  $C_{\bar{B}}^*/\bar{B}^* - r$ .

**Theorem 5** *Suppose  $0 < \tau < 1$  and  $0 < \alpha < 1$ . The optimal liability structure of a firm is unique.*

In the optimal structure,

$$\frac{\bar{V}_b^*}{V} = \bar{\pi}^{1/\lambda}, \quad (21)$$

$$\frac{\bar{B}^*}{V} = \left( \frac{1}{1-\tau} + \frac{1-\bar{\pi}}{\lambda} \right) \bar{\pi}^{1/\lambda}, \quad (22)$$

$$\frac{\bar{E}^*}{V} = 1 - \left( 1 + \frac{1-\bar{\pi}}{\lambda} \right) \bar{\pi}^{1/\lambda}, \quad (23)$$

$$\frac{\bar{F}^*}{V} - 1 = \frac{\tau}{1-\tau} \bar{\pi}^{1/\lambda}, \quad (24)$$

$$\frac{C_{\bar{B}}^*}{\bar{B}^*} - r = \frac{r\{1 + \lambda + [\tau + (1-\tau)\alpha]\lambda\}\bar{\pi}}{1 + \lambda + \{1 + \lambda + [\tau + (1-\tau)\alpha]\lambda\}\bar{\pi}}, \quad (25)$$

where  $\bar{\pi}$  is the state price of the endogenous default, which is given by

$$\bar{\pi} = \frac{\tau}{\tau(1 + \lambda) + (1 - \tau)\lambda\alpha}. \quad (26)$$

We regard the optimal liability structure in Leland's model as the optimal liability structure of the imagined bank that constrains deposits to zero.

The proposition below compares the optimal liability structure of a bank that takes deposits with the optimal structure of a bank that does not take deposits.

**Proposition 1** *Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then,*

$$\frac{V_b^*}{V} > \frac{\bar{V}_b^*}{V}, \quad \frac{T^*}{V} < \frac{\bar{T}^*}{V}, \quad \frac{E^*}{V} > \frac{\bar{E}^*}{V}, \quad \frac{F^*}{V} > \frac{\bar{F}^*}{V}, \quad \frac{C_B^*}{B^*} > \frac{C_{\bar{B}}^*}{\bar{B}^*}. \quad (27)$$

The proofs of all propositions including this one are provided in Appendix A.4. This proposition shows that the optimal liability structure of a bank that takes deposits has a higher bankruptcy boundary, a lower tangible equity ratio, a higher market equity ratio, a higher bank value, and a higher credit premium than the liability structure of a bank that does not take deposits.

Table 2 presents a numerical example of the optimal liability structure in a bank that takes deposits. The table also presents the optimal liability structure in a bank that does not take deposits. The parameters used for generating the optimal liability structure are simply for the purpose of numerical illustration, although we choose the parameters to be roughly related to the average characteristics of banks and markets in the U.S. We discuss the parameters in the appendix (section A.5) and vary the parameters later in this section. However, readers should not regard this table as a calibration of historical average of the U.S. bank liability structures for several reasons. First, the nondeposit debt in a real bank consists of many kinds of debts, ranging from short-term uninsured deposits, federal funds, and repos to long-term senior secured debts and subordinated debts. Some banks even have convertible debts or preferred equity. Second, the FDIC insurance premiums adjust for bank liability structure and

to remove subsidy only after 2011. Third, bank regulation in the U.S. has changed dramatically through the history. Therefore, Table 2 is intended to serve as a numerical illustration of our model.

| Endogenous variable | Definition      | With deposits | Definition              | No deposits |
|---------------------|-----------------|---------------|-------------------------|-------------|
| Deposits            | $D^*/V$         | 45.28         |                         |             |
| Nondeposit debt     | $B^*/V$         | 46.49         | $\bar{B}^*/V$           | 60.03       |
| Tangible equity     | $T^*/V$         | 8.24          | $\bar{T}^*/V$           | 39.97       |
| Closure boundary    | $V_a^*/V$       | 46.20         | $\bar{V}_a^*/V$         |             |
| Default boundary    | $V_d^*/V$       | 46.20         | $\bar{V}_d^*/V$         | 35.28       |
| Bankruptcy boundary | $V_b^*/V$       | 46.20         | $\bar{V}_b^*/V$         | 35.28       |
| Insurance premium   | $I^*/D^*$       | 0.53          |                         |             |
| Credit premium      | $C_B^*/B^* - 1$ | 2.27          | $\bar{C}_B^*/\bar{B}^*$ | 1.02        |
| Charter value       | $F^*/V - 1$     | 24.50         | $\bar{F}^*/V - 1$       | 6.23        |

Table 2: The optimal liability structure of an insured bank that takes deposits and the optimal liability structure of a bank that does not take deposits. The values for the endogenous variables are reported in percentage points. The key exogenous parameters are  $\eta = 0.03$ ,  $\sigma = 0.05$ ,  $\alpha = 0.27$ ,  $\beta = 0.02$ ,  $\omega = 0.1$ , and  $\tau = 0.15$ . The choice of the parameters are explained in the appendix (section A.5).

Table 2 reveals some distinctive characteristics of the optimal bank liability structure. The most striking is the high leverage in the bank that takes deposits. The optimal deposit-to-asset ratio is 45.30%, and the optimal debt-to-asset ratio is 46.49%. This liability structure leaves the tangible equity to be only 8.23% of the asset value. Another noticeable characteristic of the optimal liability structure is the significance of nondeposit debt. The optimal debt-to-asset ratio is 46.49% in the bank that takes deposits. As noted in Theorem 3, the bank optimally sets nondeposit debt to a level such that the default boundary is exactly the same as the closure boundary. With this strategy, the bank maximizes the tax deduction without further increasing the probability of bankruptcy. The bankruptcy boundary of the bank that takes deposits is 46.20% of the initial asset value. This is same as the endogenous default boundary and the regulatory closure boundary in the optimal liability structure.

To understand the effects of taking deposits, Table 2 presents in the last column the optimal liability structure of a bank that does not take deposits. Without deposits, this bank drastically reduces leverage. The tangible equity ratio in this bank is nearly 40% of the initial asset value, much higher than the tangible equity ratio in the bank that takes deposits. However, the bank that does not take deposits issues slightly more nondeposit debt than the bank that takes deposits. The bankruptcy boundary of the bank that does not take deposits is 35.28% of the initial asset value, lower than the bankruptcy boundary of the bank that takes deposits.

The higher bankruptcy boundary of the bank that takes deposits, which are 46.20% of the initial asset value, leads to a higher value of bankruptcy loss and thus a higher credit premium. The credit premium of the nondeposit debt issued by the bank that takes deposits is 227 basis points whereas the credit premium for the bank that does not take deposits is only 102 basis

points. The comparisons of the ratios and credit premiums in Table 2 illustrate the important role of deposits in bank liability structure.

Deposits allow the bank to generate a much higher charter value. In Table 2, the charter value of the bank that takes deposits is 24.50% of the assets, whereas the charter value of the bank that does not take deposits is only 6.23%. This explains why banks take deposits. This may also shed light on why bank leverage is typically higher than the leverage of nonfinancial firms.

However, banks and firms do not hold the same assets. Moody's KMV estimates that the median asset volatility of manufacturing firms ranges from 30% to 50% during 2001–2012, very different from the volatility of banks. By contrast, the median asset volatility of banks is around 5% during 2001–2012. We provide more detailed information about the estimates of Moody's KMV in Appendix A.5. If we increase the asset volatility from 5% to 30%, the tangible equity ratio in the last column of Table 2 will rise further.

The association between the low bank asset volatility and high bank leverage suggests that the low volatility is part of the endogenous choice by banks. As we have pointed out earlier, if a bank optimizes its assets and liabilities simultaneously, the observed liability structure should be optimal relative to the observed assets. In this paper, we focus on the properties of the optimal liability structure that is optimal relative the assets.

## 4.2 Effects of Individual Bank Characteristics

Different banks may face different service incomes, different asset volatilities, or different bankruptcy costs. We therefore regard the parameters  $(\eta, \sigma, \alpha)$  as characteristics of banks. Banks are heterogenous in these characteristics.

Let us first consider the effects of the service income because it is the most important reason for a bank to take deposits. The next proposition summarizes how the service income affects on the optimal bank liability structure.

**Proposition 2** *Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then, the marginal effects of the account service income  $\eta$  are*

$$\frac{\partial B^*/V}{\partial \eta} > \frac{\partial D^*/V}{\partial \eta} > 0, \quad \frac{\partial T^*/V}{\partial \eta} < 0, \quad \frac{\partial (C_B^*/B^*)}{\partial \eta} > \frac{\partial (I^*/D^*)}{\partial \eta} > 0. \quad (28)$$

The proposition makes predictions not only on the direction of the effects but also on the relative sensitivity of the financial ratios and premiums to the service income. The first chain of inequalities in (28) shows that the effect on the debt-to-asset ratio is larger than the effect on the deposit-to-asset ratio. The last chain of inequalities shows that both the insurance

premium and the credit premium are positively related to the service income rate and that the credit premium is more sensitive than the insurance premium. These may be surprising at first glance, but these are consequences of setting the endogenous default boundary and regulatory closure boundary equal in the optimal structure. We will discuss these further with the examples presented in the next figure.

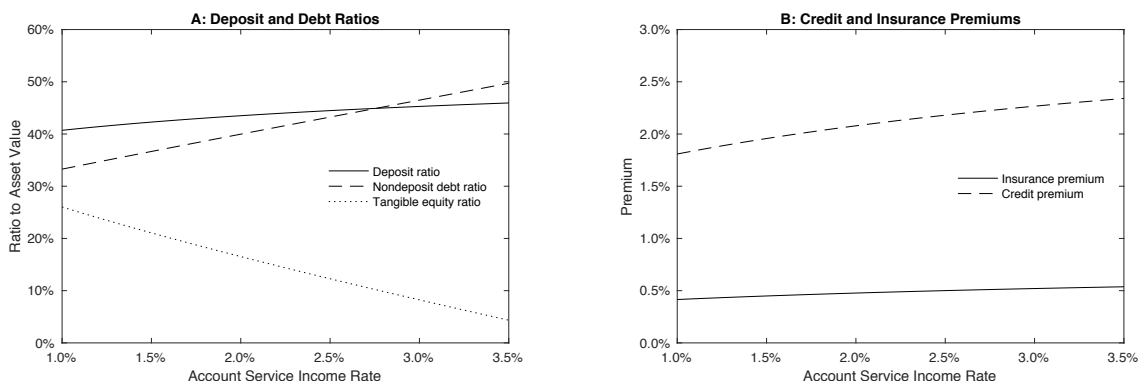


Figure 2: Effects of the account service income on the optimal structure of an insured bank. When  $\eta$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

Figure 2 illustrates the effects of the account service income. The optimal tangible equity (dotted line) in panel A decreases when the service income rate increases. If the service income rate changes from 1% to 3.5%, the tangible equity ratio decreases from about 36% to less than 5%. So the optimal leverage goes up if the service income increases. Not only the optimal deposit-to-asset ratio goes up, but the optimal debt-to-asset ratio also goes up. So an increase in the service income rate does not have a substitution effect between deposits and nondeposit debt. They both go up because the higher deposit-to-asset ratio raises the closure boundary, giving more room for the nondeposit debt. As a result, the optimal liability structure consists of more deposits and more nondeposit debt if the service income is higher.

The effects of the account service income on bank leverage is consistent with DeAngelo and Stulz (2014). They suggest that the profitability of the deposit accounts is responsible for banks' high leverage, although the banks in their paper are not covered under insurance or regulations. A major difference of our analysis from DeAngelo and Stulz's is that their banks do not use nondeposit debt to finance assets, whereas our model further predicts that the optimal nondeposit debt is also positively related to the profitability of deposit accounts. Moreover, our model predicts that the nondeposit debt ratio is more sensitive to the account service income than the deposit ratio. In panel A of Figure 2, the nondeposit debt ratio (dashed line) rises

faster than the deposit ratio (solid line).

It is interesting that both the insurance premium and the credit premium in panel B are positively related to the account service income. Before 1980, the U.S. bank regulation prohibited banks from paying interests on deposits and limited bank competition for deposits. It was thought that making deposit service more profitable would reduce the probability of a bank failure. This line of thought ignores a bank's optimal response to a change in the profitability of deposit service. Proposition 2 and Figure 2 show that a bank's optimal response is to take more deposits and to issue more nondeposit debt when deposit service becomes more profitable. This response raises the closure boundary, which increases the probability of a bank failure. Then, the expected bankruptcy loss is higher, and the risk premium is higher. Also notice that the credit premium (dashed line) in panel B of Figure 2 rises faster than the insurance premium (solid line), as predicted by Proposition 2. These complicated consequences underscore the importance to understand the balance between deposits and nondeposit debt in the optimal bank liability structure.

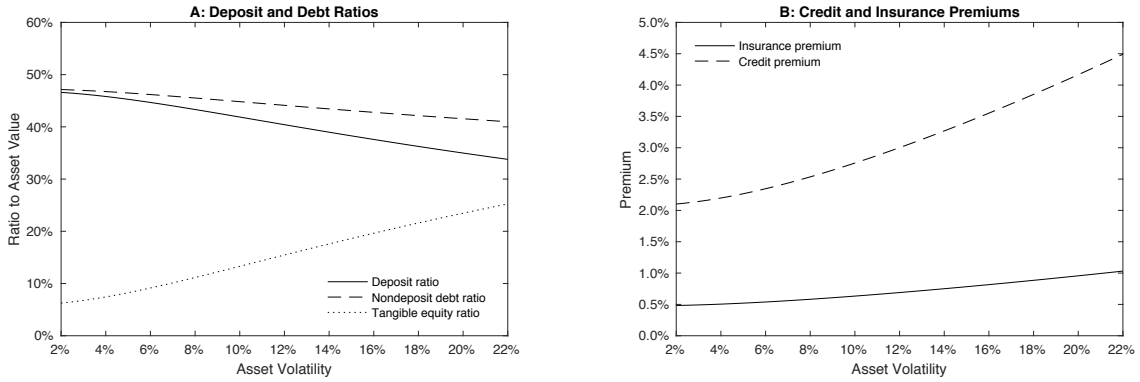


Figure 3: Effects of the asset volatility on the optimal liability structure of an insured bank. When  $\sigma$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

Bank asset volatility is typically lower than nonfinancial firms, as we have noted earlier, and the low asset volatility is important for banks to use significant leverage. The next proposition summarizes how the asset volatility affects the optimal bank liability structure.

**Proposition 3** Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then, the marginal effects of the asset volatility  $\sigma$  are

$$\frac{\partial D^*/V}{\partial \sigma} < \frac{\partial B^*/V}{\partial \sigma} < 0, \quad \frac{\partial T^*/V}{\partial \sigma} > 0, \quad \frac{\partial (C_B^*/B^*)}{\partial \sigma} > \frac{\partial (I^*/D^*)}{\partial \sigma} > 0. \quad (29)$$

The positive effects of the asset volatility on the tangible equity ratio, the credit premium, and the insurance premium are all intuitive. The negative effects on the deposits and the nondeposit debt are also expected. In addition, the proposition predicts that the deposit-to-asset ratio is more sensitive to the asset volatility than the debt-to-asset ratio, while the credit premium is more sensitive than the insurance premium.

Figure 3 illustrates the effects of asset volatility on the optimal bank liability structure. If the volatility is as low as 2%, the optimal tangible equity ratio (dotted line) in panel A is about 6%. If the asset volatility is 22%, the tangible equity ratio is above 25%. In view of the negative relationship between volatility and leverage, a manufacturing company with an asset volatility higher than 30% or 40% is unlikely to set the leverage as high as a bank's leverage.

A low asset volatility is crucial for a bank that takes significant deposits. Panel A of Figure 3 shows that a bank with an asset volatility above 20% has to take fewer deposits than 35% of the asset value. The figure also shows that the deposit ratio is more sensitive to the asset volatility than the nondeposit debt ratio. Thus, a bank needs a low asset volatility so that it is optimal to take significant deposits. Only if the optimal liability structure consists of significant deposits, the service on deposits can be a major business of the bank.

Bankruptcy cost is another key factor in optimal liability structure. The next proposition summarizes the effects of the bankruptcy cost.

**Proposition 4** *Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then, the marginal effects of the bankruptcy cost  $\alpha$  are*

$$\frac{\partial D^*/V}{\partial \alpha} < \frac{\partial B^*/V}{\partial \alpha} < 0, \quad \frac{\partial T^*/V}{\partial \alpha} > 0, \quad \frac{\partial C_B^*/B^*}{\partial \alpha} < 0, \quad \frac{\partial I^*/D^*}{\partial \alpha} > 0. \quad (30)$$

The proposition shows that the optimal deposit-to-asset ratio is more sensitive to the bankruptcy cost than the debt-to-asset ratio. This appears counterintuitive since the deposits are insured while the nondeposit debt suffers from bankruptcy cost. However, a rise in the bankruptcy cost raises the endogenous deposit insurance premium, causing the bank to reduce the deposit-to-asset ratio. In addition, the bank needs to reduce the bank leverage while still keeping the regulatory closure boundary and the endogenous default boundary equal, causing a further reduction in the deposit-to-asset ratio.

Figure 4 illustrates the effects of bankruptcy cost on the optimal bank liability structure. When we vary  $\alpha$  in the range from 15% to 40%, both the deposits and the nondeposit debt vary inversely with the bankruptcy cost. If the bankruptcy costs less, the tangible equity ratio is lower and the leverage is higher. The elevated leverage has a consequence in the credit premium. Without considering the optimal response, one would expect the credit premium to be lower if the bankruptcy cost is lower. To the contrary, the credit premium is higher



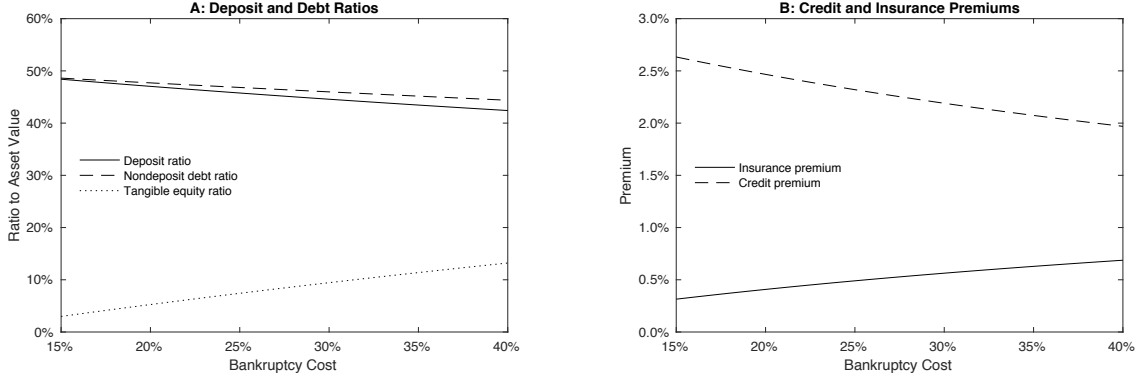


Figure 4: Effects of the bankruptcy cost on the optimal structure of an insured bank. When  $\alpha$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

for a smaller  $\alpha$ . The apparently counterintuitive relation between the credit premium and the bankruptcy cost is the result of both the optimal increase in leverage and the optimal adjustment of the relative ratio of the deposits to the nondeposit debt.

### 4.3 Effects of Regulation, Insurance Policy, and Tax

The banks are regulated, insured, and taxed in our model. The parameters  $(\beta, \omega, \tau)$  represent the environment of regulation, insurance policy, and taxation for banks. Recall that  $\beta$  represents the minimum capital requirement,  $\omega$  represents the insurance subsidy, and  $\tau$  represents the corporate tax rate. The regulation, insurance policy, and taxation determine these parameters. These parameters change when the regulation, the insurance policy, and tax rules change. A change in the regulation, insurance policy, or the tax rules usually affect banks' optimal choice of liability structure.

Let us first examine the effects of changes in the minimum capital requirement  $\beta$ . The next proposition summarizes the effects.

**Proposition 5** *Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then, the marginal effects of the minimum capital requirement  $\beta$  are*

$$\frac{\partial D^*/V}{\partial \beta} < 0, \quad \frac{\partial B^*/V}{\partial \beta} > 0, \quad \frac{\partial T^*/V}{\partial \beta} > 0, \quad \frac{\partial I^*/D^*}{\partial \beta} < \frac{\partial C_B^*/B^*}{\partial \beta} < 0. \quad (31)$$

Proposition 5 says that tightening the minimum capital requirement reduces the deposits and the leverage. Figure 5 provides an illustration of the effects. In the figure, we vary  $\beta$

from 0% to 25% and find that the optimal tangible equity ratio increases from about 8% to above 15%. The reduction in leverage is the result of the drop in the deposit-to-asset ratio from about 46% to nearly 33%. The reduction in leverage lowers both the credit premium and the insurance premium, as we see in Proposition 5 and in panel B of Figure 5. These effects of  $\beta$  are as expected.

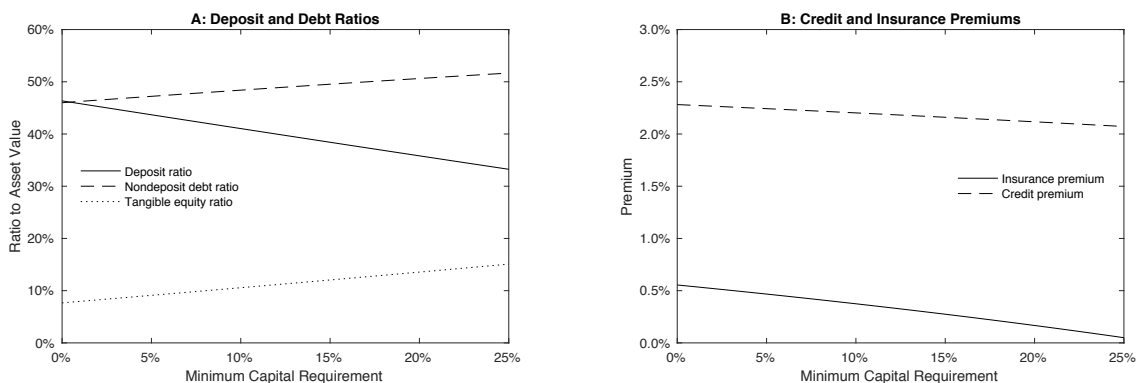


Figure 5: Effects of the minimum capital requirement on the optimal structure of an insured bank. When  $\beta$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

However, the effect of the capital requirement on the nondeposit debt is worth to pay a special attention. The effect is positive in the proposition. In the table, increasing  $\beta$  from 0% to 25% causes the bank to increase the nondeposit debt from about 46% to above 51% of the asset value. This is again resulted from setting the endogenous default boundary and the regulatory closure boundary equal in the optimal structure. If the liability structure is kept fixed, an increase in  $\beta$  raises only the regulatory closure boundary but not the endogenous default boundary. Meanwhile, the decrease in the deposits lowers both the closure boundary and the endogenous default boundary. The bank needs to issue more nondeposit debt to bring the endogenous default boundary up to the same level of the regulatory closure boundary so that its liability structure remains to be optimal. This effect again shows the importance of the equal boundaries in understanding the optimal bank liability structure.

In fact, the bankruptcy boundary decreases if regulators tighten the minimum capital requirement. This inverse relation between the bankruptcy boundary and the minimum capital requirement is also caused by the optimal response of the bank liability structure. For a higher  $\beta$ , a bank takes fewer deposits. The drop in deposits is so large that it completely counteracts the increase in  $\beta$ . Figure 6 illustrates how the optimal response complicates the relation between the closure boundary and the minimum capital requirement. In the figure,

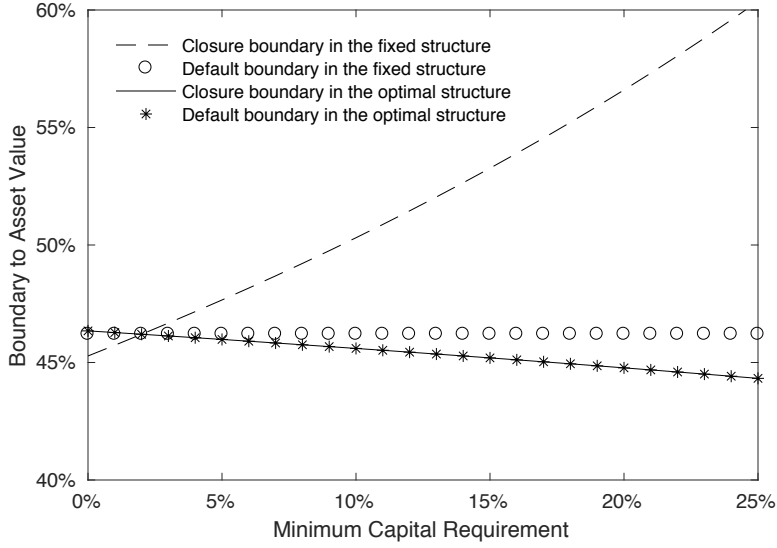


Figure 6: The bankruptcy boundary of an insured bank. The bank first optimizes its liability structure for the parameters used in Table 2. When the minimum capital requirement  $\beta$  varies in the range from 0% to 25%, the liability structure is first kept fixed and then re-optimized for each changed  $\beta$ . The panel plots the closure boundary  $V_a/V$  and the endogenous default boundary  $V_d/V$  for each value of  $\beta$  in the fixed liability structure and then plots  $V_a^*/V$  and  $V_d^*/V$  in the re-optimized liability structure.

we first optimize the liability structure of the bank for  $\beta = 2\%$ , and then we let  $\beta$  change. If we do not optimally adjust the liability structure for the change in  $\beta$ , the closure boundary,  $V_a = D/(1 - \beta)$ , should be an increasing function of  $\beta$  plotted as the dashed line in the figure, whereas the default boundary  $V_d$  should be independent of  $\beta$ , as marked by the circles.

If we optimally adjust the liability structure to the change in  $\beta$ , however, the relation between the closure boundary and  $\beta$  is completely different; the closure boundary decreases as  $\beta$  increases. As  $\beta$  increases from 0% to 25%, the bankruptcy boundary gets lower, dropping from above 46% to below 45% of the initial asset value. The closure boundary  $V_a^* = D^*/(1 - \beta)$  in the optimal liability structure is not a complicated function of  $\beta$  because the bank optimally reduces deposits  $D^*$  in response to the increase in  $\beta$ . The endogenous default boundary  $V_d^*$  is related to  $\beta$  the same way because  $V_d^*$  has to be equal to  $V_a^*$  for all  $\beta$ .

In all the analyses so far in this paper, we assume that the subsidy in the insurance premium is  $\omega = 10\%$ . The next proposition considers how a change in the insurance subsidy affects the optimal bank liability structure.

**Proposition 6** Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). Then, the marginal

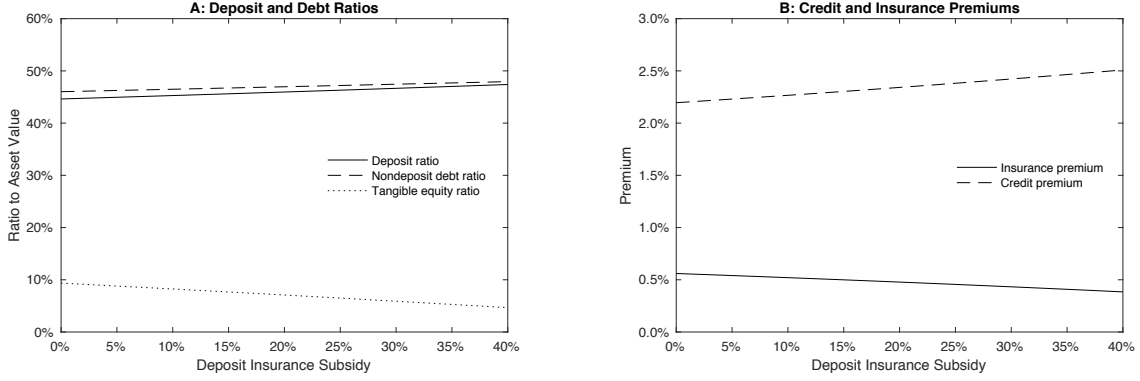


Figure 7: Effects of the insurance subsidy on the optimal structure of an insured bank. When  $\omega$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

effects of the insurance subsidy  $\omega$  are

$$\frac{\partial D^*/V}{\partial \omega} > \frac{\partial B^*/V}{\partial \omega} > 0, \quad \frac{\partial T^*/V}{\partial \omega} < 0, \quad \frac{\partial C_B^*/B^*}{\partial \omega} > 0, \quad \frac{\partial I^*/D^*}{\partial \omega} < 0. \quad (32)$$

The proposition shows that with a larger insurance subsidy, a bank takes more deposits and issues more nondeposit debt. The proposition also shows that the effect on the deposits is larger than the effect on the nondeposit debt. The insurance subsidy reduces the insurance premium but makes banks more leveraged, resulting in a higher credit premium.

Figure 7 illustrates Proposition 6. We consider various magnitudes for the subsidy, ranging from zero to 40%, and look at its effects on the liability structure. Clearly, an insurance subsidy encourages bank leverage. As the subsidy increases from zero to 40%, the tangible equity ratio drops from about 9% to about 5%. The subsidy not only increases the optimal deposit-to-asset ratio but also increases the optimal debt-to-asset ratio. While the deposit-to-asset ratio goes up from about 45% to about 47%, the debt-to-asset ratio rises from about 46% to about 47%. Meanwhile, the credit premium rises. As expected, though, the insurance premium drops if the insurance is more subsidized.

Our model offers a framework to examine the link between tax policy and bank leverage. The next proposition shows how the corporate tax influences the optimal bank liability structure.

**Proposition 7** Suppose  $0 < \eta < r$ ,  $0 < \tau < 1$ ,  $\epsilon < \beta < 1$ , and  $\omega \in [0, 1)$ , where  $\epsilon$  is defined in Theorem 3. Suppose the insurance premium is determined by equation (11). For the marginal

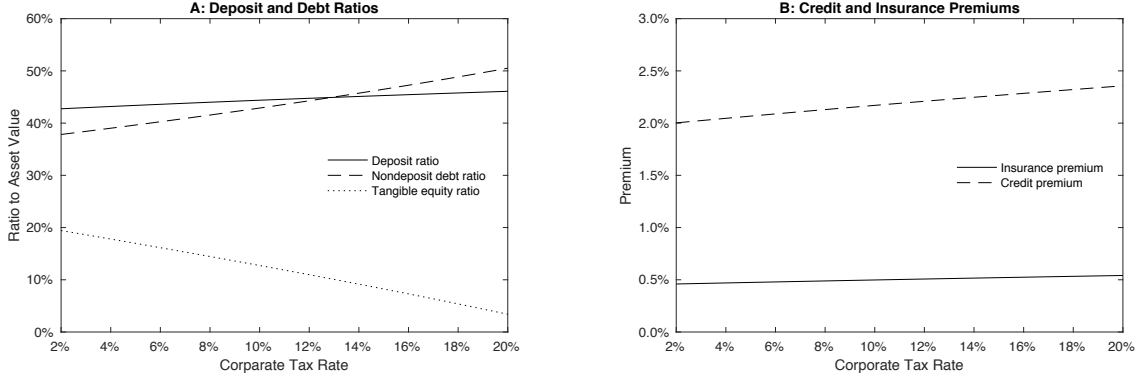


Figure 8: Effects of the corporate tax on the optimal structure of an insured bank. When  $\tau$  varies, the other parameters are fixed at the values used in Table 2. The solid line in panel A is the optimal deposits, the dashed line is the optimal nondeposit debt, and the dotted line is the optimal tangible equity, all reported as percent of assets. The solid line in panel B is the deposit insurance premium as percent of assets, and the dashed line is the credit premium of the optimal nondeposit debt.

effects of the corporate tax rate  $\tau$ , we have

$$\frac{\partial B^*/V}{\partial \tau} > \frac{\partial D^*/V}{\partial \tau} > 0, \quad \frac{\partial T^*/V}{\partial \tau} < 0, \quad \frac{\partial C_B^*/B^*}{\partial \tau} > \frac{\partial I^*/D^*}{\partial \tau} > 0. \quad (33)$$

This proposition shows that an increase in the tax rate raises both the deposit-to-asset ratio and the debt-to-asset ratio in the optimal liability structure. Moreover, the tax effect on the nondeposit debt is larger than its effect on the deposits. As expected, an increase in the tax rate causes the leverage, the bankruptcy boundary, the credit premium, and the insurance premium all to go up.

Figure 8 illustrates the optimal response of the bank liability structure to the changes in tax rate. If the tax rate is lowered from 20% to 2%, the bank reduces both the deposit-to-asset ratio and the debt-to-asset ratio. The deposit-to-asset ratio drops by less than 4 percentage point, from about 46% to 42%. By contrast, the ratio of the nondeposit debt to assets reduces by nearly 12 percentage points. Since the nondeposit debt does not bring service income, the reduction in the nondeposit debt is larger than the reduction in the deposits. Overall, the bank uses less leverage in a regime with a lower tax rate. In Figure 8, the tangible equity ratio is up by more than 16 percentage points when the tax rate is lowered from 20% to 2%. Because the leverage drops, the credit premium falls when the tax rate is lowered.

The association of lower leverage with a lower tax rate appears to support those who argue for lowering corporate taxes to stabilize banks, but we need to be cautious about this policy proposal. Since lowering corporate tax leads to loss in tax revenue, it can be an expensive policy change for the public although it helps for improving the stability of the banking industry. An alternative is to lower corporate taxes just for banks as suggested by Fleischer (2013). This

proposed policy will make banking a subsidized business, begging questions on the fairness of the corporate tax policy and questions on the distortions to efficient resource allocation in the economy.

Those important issues are beyond the scope of this paper, but the dependence of bank liability structure on the tax rate described in our model lays a stepping-stone for careful analyses of the benefit and cost of tax policy reforms. As noted earlier, Schepens (2014) exploits a recent tax code change that permits some tax advantages to equity and shows that banks do respond to the changes by altering their liability structure. Our model suggests that the analysis of tax policy must not ignore banks' optimal responses. Along the line of this idea, Gambocorta et al. (2016) examine how Italian community banks optimally respond to the changes in the tax rates for those banks.

## 5 Conclusion

Our theory of bank liability structure explicitly incorporates an array of factors that affect the optimal leverage and the liability composition of banks. The factors include the service income from deposits, the FDIC insurance, and the minimum capital requirement, besides the asset volatility, the bankruptcy cost, and the corporate tax, which are the factors that affect the liability structure of firms.

Since our model is structural and solved analytically, it provides a salient feature of bank liability structures: a bank uses as much nondeposit debt as possible to take advantage of the tax benefits but not as much to offer unwarranted protection for deposits. This salient feature is likely to be robust if bank assets are optimized simultaneously with the liability structure because the liability structure should be optimal relative to the bank assets. Our comparative static analysis shows that that salient feature is a key element in understanding how the factors affect bank liability structures.

Models that characterized by a few time steps and discrete asset values are common in banking studies. Those models are useful for certain conceptual issues but do not have the salient feature of our model.

Our model of bank liability structure provides tools for studying some important issues. For example, some academics propose alternative bank liability structures that include securities beyond deposits and debt (Flannery, 2009). A controversial security since the crisis of 2007–2009 is contingent capital, which is a debt that converts to equity when the bank becomes under-capitalized. Sundaresan and Wang (2015) analyze the complex issues in designing a contingent capital. An extension of our model to include convertible debt can shed light on the interactions of the convertible debt with deposits, the traditional debt, and the deposit

insurance. These interactions are important for figuring out whether convertible debt can help stabilize banks. Another important component in bank capital structure is the liquidity reserves. Hugonnier and Morellec (2015) use a variant of our model to examine the optimal liquidity reserves in a bank. As we have noted earlier, our model is suitable for studying the effects of tax policy changes on banks. Gambocorta et al. (2016) apply a variant of our model to study the optimal responses of Italian community banks to the changes in their tax rates.

Our model may be used to shed light on the regulatory treatment of long-term debt. If a long-term debt is a claim ranked lower than the deposits, it is naturally viewed as a capital that protects the deposits. Reflecting this view, regulators treat certain long-term unsecured debt as Tier 2 regulatory capitals. However, if a bank adjusts its liability structure so that the endogenous default coincides with the regulatory closure, the long-term debt held by the bank do not offer more protection than the minimum capital requirement. A large body of academic literature debates whether long-term debt provide a market discipline on bankruptcy risk. Flannery and Serescu (1996) contend that the debt price rationally reflects the risk in a bank. Gorton and Santomero (1990), however, opine the opposite. The optimal choice of nondeposit debt in a bank liability structure is especially relevant to this debate.

Our model can be extended to a setting in which banks dynamically change the liability structures as well as its composition of assets. This is especially important for banks because they can take more deposits and raise leverage to increase bank value. In reality, this prospect is not always available as the supply of deposits is finite. Furthermore, in a depressed economy, the risk-free rate itself can be very low, limiting the ability of banks to gain from this channel. Goldstein et al. (2001) have developed a dynamic framework for a corporate borrower. In their model, firms consider the opportunity of issuing additional debt when they optimize capital structure. By contrast, an important issue for a bank is the ability to reduce leverage when the bank becomes poorly capitalized after large losses. Another important issue for a bank is that it can dynamically adjust the asset structure in response to changes in risk. Adrian and Shin (2010) document that financial intermediaries adjust balance sheets in response to changes in their forecasts of risk. Extension of our model to such a dynamic setting will be a useful, but challenging, project that warrants further research.

## A Appendix

### A.1 Proof of Theorem 1

The general solution to pricing equation (3) is

$$E = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + V - (1 - \tau)(I + C_D + C_B)/r \quad (34)$$

where  $\lambda > 0$  and  $\lambda' < 0$  are the two solutions to equation (4), and  $a_1$  and  $a_2$  are arbitrary constants. The boundary conditions of  $E$  imply  $a_2 = 0$  and  $a_1 = -[V_b - (1 - \tau)(I + C_D + C_B)/r]V_b^\lambda$ , which give equation (8).

If  $V_b = V_a = D/(1 - \beta)$ , then  $C_D = (r - \eta)D$  gives equation  $V_a = C_D/[(r - \eta)(1 - \beta)]$ . To prove

$$V_d = (1 - \tau) \frac{\lambda}{1 + \lambda} \cdot \frac{I + C_D + C_B}{r} \quad (35)$$

is the endogenous default boundary, we need to show that  $V_b$  maximizes the equity value when  $V_b = V_d$ . Differentiation of equation (8) with respect to  $V_b$  leads to

$$\partial E / \partial V_b = [(1 + \lambda)/V_b](V_b/V)^\lambda (V_d - V_b). \quad (36)$$

Since the above is positive if  $V_b < V_d$  and negative if  $V_b > V_d$ , we know  $V_b = V_d$  maximizes the equity value. Notice that  $V_d$  is independent of  $V$ . Equity holders choose to default before bank closure if  $V$  drops to  $V_d$  before  $V_a$ . The bank is closed before endogenous default if  $V$  drops to  $V_a$  first. Therefore, the bankruptcy boundary is  $V_b = \max\{V_a, V_d\}$ .

The general form of solution to pricing equation (2) is

$$B = a_1 V^{-\lambda} + a_2 V^{-\lambda'} + C_B/r, \quad (37)$$

where  $a_1$  and  $a_2$  can be any constants. The boundary conditions of  $B$  imply  $a_2 = 0$  and  $a_1 = \{[(1 - \alpha)V_b - D]^+ - C_B/r\}V_b^\lambda$ , which give equation (7).

Bank value is  $F = D + B + E$ . We obtain equation (9) by substituting equations (6), (7), and (8).

## A.2 Proof of Theorem 2

Let  $Q$  be the value of deposit insurance to banks. Its pricing equation is

$$\frac{1}{2}\sigma^2 V^2 Q'' + (r - \delta)VQ' - rQ - I = 0, \quad (38)$$

where  $Q'$  and  $Q''$  denote the first and second partial derivatives of  $Q$  with respect to  $V$ . The general solution to the equation is  $Q(V) = -I/r + a_1 V + a_2 V^{-\lambda}$ , where  $a_1$  and  $a_2$  can be any constants. The boundary conditions of the value of the insurance product are  $\lim_{V \rightarrow \infty} Q = -I/r$  and  $Q(V_a) = [D - (1 - \alpha)V_a]^+$ , where  $V_a = D/(1 - \beta)$ . They imply  $a_1 = 0$  and  $-I/r + a_2 V_a^{-\lambda} = [D - (1 - \alpha)V_a]^+$ , which give  $Q(V) = -(1 - P_a)I/r + [D - (1 - \alpha)V_a]^+ P_a$ , where  $P_a = [V_a/V]^\lambda$ . The insurance premium  $I^\circ$  is fair if and only if  $Q(V) = 0$ . It follows that  $(1 - P_a)I^\circ = r[D - (1 - \alpha)V_a]^+ P_a$ . We obtain equation (10) by substituting  $V_a = D/(1 - \beta)$  and factoring  $D$  out.



### A.3 Proof of Theorem 3

To simplify the derivation of optimal liability structure, we introduce the following notations:

$$x = C_B/C_D, \quad c = C_D/(rV), \quad v_a = rV_a/C_D, \quad v_d = rV_d/C_D, \quad v_b = rV_b/C_D, \quad (39)$$

$$\iota = \eta/(r - \eta), \quad \theta = (1 - \tau)\lambda/(1 + \lambda), \quad i = I/C_D, \quad \kappa = 1/(1 - \beta). \quad (40)$$

We refer to  $x$  as the liability ratio and  $c$  as the deposit liability scaled by asset value. The state price of bankruptcy is then simplified to  $P_b = (v_b c)^\lambda$ . By Theorem 1, the scaled boundaries are

$$v_a = \kappa(1 + \iota), \quad v_d = \theta(1 + i + x), \quad v_b = \max\{\kappa(1 + \iota), \theta(1 + i + x)\}. \quad (41)$$

Notice that  $V_a < V_d$  if and only if  $v_a < v_d$ . Furthermore, equation (11) implies

$$i = (1 - \omega)[1 - (1 - \alpha)\kappa]^+(1 + \iota)(v_a c)^\lambda/[1 - (v_a c)^\lambda]. \quad (42)$$

We can also express the bank value in Theorem 1 as a ratio to asset value:

$$\begin{aligned} f(x, c) = F/V = 1 + [1 - (v_b c)^\lambda][\iota + \tau(1 + i + x) - i]c \\ - (v_b c)^\lambda \min\{\alpha v_b, v_b - (1 + \iota)\}c. \end{aligned} \quad (43)$$

Choosing  $(C_D, C_B)$  to maximize bank value  $F$  is equivalent to choosing the duplet  $(x, c)$  to maximize  $f$ . Once we obtain the optimal  $(x^*, c^*)$ , the optimal liabilities  $(C_D^*, C_B^*)$  can be obtained easily as  $C_D^* = c^* rV$  and  $C_B^* = x^* C_D^*$ .

We first show that  $v_d < v_a$  is not optimal. If  $v_d < v_a$ , then  $\theta(1 + i + x) < (1 + \iota)\kappa$ ,  $v_b = v_a = (1 + \iota)\kappa$ , and

$$f(x, c) = 1 + c \left\{ [\iota - i + \tau(1 + i + x)][1 - (v_a c)^\lambda] - (\kappa - 1)(1 + \iota)(v_a c)^\lambda \right\}. \quad (44)$$

We then obtain  $f'_x(x, c) = \tau[1 - (v_a c)^\lambda]c > 0$ , which implies that the current  $x$  is not optimal.

It follows from equation (42) that  $i'_c = \partial i / \partial c = \lambda i c^{-1} / [1 - (v_a c)^\lambda]$ . Since  $\kappa < 1/(1 - \alpha)$ , both  $i$  and  $i'_c$  are positive. We also have  $c i'_c [1 - (v_b c)^\lambda] \leq \lambda i$  because  $v_b \geq v_a$ . The equality,  $c i'_c [1 - (v_b c)^\lambda] = \lambda i$  holds when  $v_b = v_a$ . Both  $i$  and  $i'_c$  converge to zero when  $\kappa$  rises to  $1/(1 - \alpha)$  while other parameters and variables are fixed. Thus, given any  $v > v_a$ , there exists  $\kappa^* \in [1, 1/(1 - \alpha))$  such that  $\kappa \in (\kappa^*, 1/(1 - \alpha))$  implies  $i + c i'_c < \iota$  for all  $c$  in  $[0, 1/v]$ . Let  $\epsilon = (\kappa^* - 1)/\kappa^*$ . Then,  $\kappa \in (\kappa^*, 1/(1 - \alpha)]$  for all  $\beta \in (\epsilon, \alpha]$ .

If  $v_d > v_a$ , we have  $\theta(1 + i + x) > (1 + \iota)\kappa$  and  $v_b = \theta(1 + i + x)$ . Notice that

$$\min\{\alpha v_b, v_b - (1 + \iota)\} = \begin{cases} v_b - (1 + \iota) & \text{if } v_b \leq (1 + \iota)/(1 - \alpha) \\ \alpha v_b & \text{if } v_b > (1 + \iota)/(1 - \alpha). \end{cases} \quad (45)$$

If  $v_a < v_d \leq (1 + \iota)/(1 - \alpha)$ , equations (45) and (43) give

$$f'_x(x, c) = c \left[ \tau - (\tau + \lambda)(v_d c)^\lambda + \lambda(v_d c)^\lambda \frac{1 + i}{1 + i + x} \right] \quad (46)$$

$$f'_c(x, c) = 1 + \iota - (1 + i + ci'_c) \left[ 1 - (v_d c)^\lambda \right] \\ + (1 + i + x + ci'_c) \left\{ \tau - (\tau + \lambda)(v_d c)^\lambda + \lambda(v_d c)^\lambda \frac{1 + i}{1 + i + x} \right\}. \quad (47)$$

Let  $c_x$  be the optimal  $c$  for given  $x$ , then equation (47) implies

$$\tau - (\tau + \lambda)(v_d c_x)^\lambda + \lambda(v_d c_x)^\lambda \frac{1 + i}{1 + i + x} = - \frac{1 + \iota - (1 + i + c_x i'_c) [1 - (v_d c_x)^\lambda]}{1 + i + x + c_x i'_c}. \quad (48)$$

For  $\kappa \in (\kappa^*, 1/(1 - \alpha))$ , we have  $i + c_x i'_c < \iota$ , which implies that the numerator is positive. Substitution of the above into equation (46) shows  $f'_x(x, c_x) < 0$ . Thus,  $v_a < v_d < (1 + \iota)/(1 - \alpha)$  is not optimal because lowering  $x$  and  $v_d$  increases  $f(x, c_x)$ .

If  $v_b \geq (1 + \iota)/(1 - \alpha)$ , equation (45) and (43) give

$$f'_x(x, c) = c \left\{ \tau - [(1 + \lambda)(\tau + \alpha\theta) + \lambda(\iota - i)/(1 + i + x)] (v_d c)^\lambda \right\}. \quad (49)$$

$$f'_c(x, c) = (\iota - i - ci'_c) [1 - (v_d c)^\lambda] \\ + (1 + i + ci'_c + x) \cdot \left\{ \tau - [(1 + \lambda)(\tau + \alpha\theta) + \frac{\lambda(\iota - i)}{1 + i + x}] (v_d c)^\lambda \right\}. \quad (50)$$

Let  $c_x$  be the optimal  $c$  relative to  $x$ . Then, equation (50) implies

$$\tau - (1 + \lambda)(\tau + \alpha\theta) + \frac{\lambda(\iota - i)}{1 + i + x} = - \frac{[\iota - i - c_x i'_c] [1 - (v_d c_x)^\lambda]}{1 + i + x + c_x i'_c}. \quad (51)$$

For  $\kappa > \kappa^*$ , we have  $i + ci'_c \leq \iota$ . Then, the numerator is positive. Substitution of the above into equation (49) shows  $f'_x(x, c_x) < 0$ , which means the current  $x$  and  $v_d$  is not optimal because lowering  $x$  and  $v_d$  increases  $f(x, c_x)$ .

The above two cases show that there exists  $\kappa^*$  such that for  $\kappa^* < \kappa < 1/(1 - \alpha)$ , we have  $f'_x < 0$  for all  $x$  satisfying  $\theta(1 + i + x) > (1 + \iota)\kappa$ , if  $c$  is kept to be optimal relative to  $x$ . Therefore,  $\theta(1 + i + x) > (1 + \iota)\kappa$  cannot be optimal because reducing  $x$  adds value to the bank. Consequently, the optimal  $x^*$  and  $c^*$  must satisfy  $\theta(1 + i^* + x^*) = (1 + \iota)\kappa$ , which implies  $v_d^* = (1 + \iota)\kappa$  and thus  $v_b^* = v_d^* = v_a$ .

With  $v_a^* = v_d^* = v_b^*$ , the state price of bankruptcy is:  $\pi = (v_a^* c^*)^\lambda = (v_b^* c^*)^\lambda$ . This equation implies  $v_a^* = \pi^{1/\lambda}/c^*$ . In view of equation (41), we have  $(1 + \iota)\kappa = \pi^{1/\lambda}/c^*$ . It follows that  $c^* = \pi^{1/\lambda}/[(1 + \iota)\kappa]$ ,  $i^* = (1 + \iota)[1 - (1 - \alpha)\kappa]^+ \pi/(1 - \pi)$ , and  $x^* = (1 + \iota)\{\kappa/\theta - (1 - \omega)[1 - (1 - \alpha)\kappa]^+ \pi/(1 - \pi)\} - 1$ .

Let  $x_c = v_a/\theta - (1 + i)$  for any  $c \in [1, 1/v_a]$  and  $g(c) = f(x_c, c)$ . It follows from equation (44) that

$$g(c) = 1 + c \left\{ [\iota - i + \tau v_a/\theta] [1 - (v_a c)^\lambda] - (\kappa - 1)(1 + \iota)(v_a c)^\lambda \right\}. \quad (52)$$

This function is differentiable in  $c$ , and

$$g'(c) = [\iota - i + \tau v_a / \theta][1 - (1 + \lambda)(v_a c)^\lambda] - (\kappa - 1)(1 + \iota)(1 + \lambda)(v_a c)^\lambda - c i'_c [1 - (v_a c)^\lambda]. \quad (53)$$

With equation (42) and the formula of  $i'_c$ , the above simplifies to

$$g'(c) = \iota + \tau v_a / \theta - \{ \iota + \tau v_a / \theta + (\kappa - 1)(1 + \iota) + (1 - \omega)[1 - (1 - \alpha)\kappa]^+(1 + \iota) \} (1 + \lambda)(v_a c)^\lambda. \quad (54)$$

If  $(x^*, c^*)$  is a maximum,  $c^*$  must maximize  $g(c)$ , and thus  $g'(c^*) = 0$ . Letting  $\pi = (v_a c^*)^\lambda$  and setting equation (54) to zero, we obtain

$$\pi = \frac{1}{1 + \lambda} \cdot \frac{\iota \theta + \tau(1 + \iota)\kappa}{\iota \theta + \tau(1 + \iota)\kappa + (1 + \iota)\{ \kappa - 1 + (1 - \omega)[1 - (1 - \alpha)\kappa]^+ \} \theta}. \quad (55)$$

Finally, we complete the proof by substituting the original parameters into (55) and the original variables into the formulas for  $c^*$ ,  $i^*$ , and  $x^*$ .

## A.4 Proofs of Propositions

Proposition 1 follows immediately from Theorem 4 and Theorem 5.

From equation (4), it is straightforward to calculate the partial derivatives of  $\lambda$  with respect to  $\sigma$ . We obtain

$$\frac{\partial \lambda}{\partial \sigma} = -\frac{\sigma \lambda^2 (1 + \lambda)}{\frac{1}{2} \sigma^2 \lambda^2 + r} < 0. \quad (56)$$

From equation (12), we can obtain

$$\frac{\partial \pi}{\partial \lambda} = -\frac{\pi}{1 + \lambda} \left[ 1 + \frac{r^2 \tau (1 - \tau) [(1 - \omega)\alpha + \omega\beta] (1 + \lambda)^2 \pi}{[r\tau(1 + \lambda) + \eta(1 - \tau)(1 - \beta)\lambda]^2} \right]. \quad (57)$$

It follows that

$$\frac{\partial \pi}{\partial \sigma} = \frac{\pi}{1 + \lambda} \left[ 1 + \frac{r^2 \tau (1 - \tau) [(1 - \omega)\alpha + \omega\beta] (1 + \lambda)^2 \pi}{[r\tau(1 + \lambda) + \eta(1 - \tau)(1 - \beta)\lambda]^2} \right] \frac{\sigma \lambda^2 (1 + \lambda)}{\frac{1}{2} \sigma^2 \lambda^2 + r} > 0. \quad (58)$$

Since the partial derivative of  $\pi^{1/\lambda}$  is

$$\frac{\partial \pi^{1/\lambda}}{\partial \lambda} = \frac{\pi^{1/\lambda}}{\lambda^2} \left[ \frac{\lambda}{\pi} \frac{\partial \pi}{\partial \lambda} - \ln(\pi) \right], \quad (59)$$

we can substitute equation (58) into the above to obtain

$$\frac{\partial \pi^{1/\lambda}}{\partial \lambda} = \frac{\pi^{1/\lambda}}{\lambda^2} \left[ \ln\left(\frac{1}{\pi}\right) - \frac{\lambda}{1 + \lambda} \left( 1 + \frac{r^2 \tau (1 - \tau) [(1 - \omega)\alpha + \omega\beta] (1 + \lambda)^2 \pi}{[r\tau(1 + \lambda) + \eta(1 - \tau)(1 - \beta)\lambda]^2} \right) \right]. \quad (60)$$

Using the inequality:  $\ln(1 + x) < x$ , which holds for  $x > -1$  and  $x \neq 0$ , we obtain  $\ln(\pi) < \pi - 1$  and thus  $\ln(1/\pi) > 1 - \pi$ . Then,

$$\frac{\partial \pi^{1/\lambda}}{\partial \lambda} \geq \frac{\pi^{1/\lambda}}{\lambda^2} \left[ 1 - \pi - \frac{\lambda}{1 + \lambda} \left( 1 + \frac{r^2 \tau (1 - \tau) [(1 - \omega)\alpha + \omega\beta] (1 + \lambda)^2 \pi}{[r\tau(1 + \lambda) + \eta(1 - \tau)(1 - \beta)\lambda]^2} \right) \right]. \quad (61)$$

Substituting  $\pi$  by equation (12), we obtain

$$\frac{\partial \pi^{1/\lambda}}{\partial \lambda} \geq \frac{\pi^{1/\lambda}}{(1+\lambda)^2} \cdot \frac{\eta(1-\tau)^2(1-\beta)r[(1-\omega)\alpha + \omega\beta]\lambda}{\Phi \cdot \{\Phi + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda}} > 0, \quad (62)$$

where  $\Phi$  is defined by

$$\Phi = r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda. \quad (63)$$

It follows that  $\partial \pi^{1/\lambda} / \partial \lambda > 0$  and thus

$$\frac{\partial \pi^{1/\lambda}}{\partial \sigma} = -\frac{\partial \pi^{1/\lambda}}{\partial \lambda} \cdot \frac{\sigma\lambda^2(1+\lambda)}{\frac{1}{2}\sigma^2\lambda^2 + r} < 0. \quad (64)$$

Also from equation (12), we directly obtain

$$\frac{\partial \pi}{\partial \eta} = \frac{r\lambda}{1+\lambda} \cdot \frac{r(1-\tau)^2(1-\beta)[(1-\omega)\alpha + \omega\beta]\lambda}{\{r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda^2}} > 0 \quad (65)$$

$$\frac{\partial \pi}{\partial \alpha} = -\frac{r\lambda}{1+\lambda} \cdot \frac{(1-\tau)(1-\omega)[r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda]}{\{r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda^2}} < 0 \quad (66)$$

$$\frac{\partial \pi}{\partial \beta} = -\frac{r\lambda}{1+\lambda} \cdot \frac{(1-\tau)\{r\omega(1-\tau)(1+\lambda) + \eta[\omega + (1-\omega)\alpha]\lambda\}}{\{r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda^2}} < 0 \quad (67)$$

$$\frac{\partial \pi}{\partial \omega} = \frac{r\lambda}{1+\lambda} \cdot \frac{(1-\tau)(\alpha - \beta)[r\tau(1+\lambda) + \eta(1-\beta)\lambda]}{\{r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda^2}} > 0 \quad (68)$$

$$\frac{\partial \pi}{\partial \tau} = \frac{r^2[(1-\omega)\alpha + \omega\beta]\lambda}{\{r\tau(1+\lambda) + \eta(1-\tau)(1-\beta)\lambda + r(1-\tau)[(1-\omega)\alpha + \omega\beta]\lambda^2}} > 0. \quad (69)$$

It follows from the above equations that

$$\frac{\partial \pi^{1/\lambda}}{\partial \eta} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \eta} > 0 \quad (70)$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \alpha} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \alpha} < 0 \quad (71)$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \beta} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \beta} < 0 \quad (72)$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \omega} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \omega} > 0 \quad (73)$$

$$\frac{\partial \pi^{1/\lambda}}{\partial \tau} = \frac{\pi^{1/\lambda}}{\lambda\pi} \cdot \frac{\partial \pi}{\partial \tau} > 0. \quad (74)$$

Then, we obtain Propositions 2–33 using Theorem 4 and the partial derivatives of  $\pi$  and  $\pi^{1/\lambda}$  that we have just derived.

## A.5 Choice of Exogenous Parameters

As our model inherits the major advantage of structural models that coherently connects the risk of debt and equity to the risk of assets, the risk comes from asset volatility ( $\sigma$ ), which is

one of the most important parameters to affect bank leverage and liability structure. Since asset volatility is not directly observable, asset volatility is typically inferred from accounting data and market prices. Moody's KMV provides estimates of asset volatility for a large number of companies across a wide range of industries.

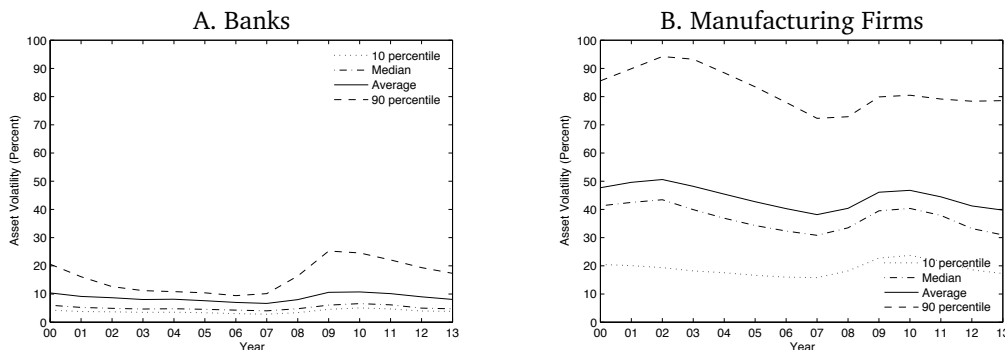


Figure 9: Plots of the average, median, and 10- and 90-percentiles of asset volatilities of banks (panel A) and manufacturing firms (panel B) from 2000 to 2013. Moody's KMV Investor Service provided the estimates of asset volatilities.

In panel A of Figure 9, we present the average, median, and the 10- and 90-percentiles of Moody's estimates of bank asset volatilities. As a comparison, in panel B we present Moody's estimates of manufacturing-firm asset volatilities. The figure shows a difference between the assets held by banks and those owned by manufacturing firms: bank assets have much lower volatility. The average asset volatility is around 10% for banks, whereas it is 40 ~ 50% for manufacturing firms. Although bank asset volatility fluctuates over time, the median is around 5% for 2001–2012. The 90 percentile of bank asset volatilities is well below 15% for 2001–2007, and it stays below 25% even for the period of 2007–2012. In view of these facts, we let  $\sigma \in [2\%, 22\%]$  in our study of comparative statics.

Another parameter of bank assets is its rate of cash flows ( $\delta$ ). If the assets contain only commercial and consumer loans, the cash flows are interest and principal payments of the loans. In the numerical illustration and comparative statics, we set the cash-flow rate to 8%, which is the average mortgage rate in the U.S. during 1984–2013. Correspondingly, we also set the risk-free rate to the average federal funds rate during the same period; this gives  $r = 5\%$ . We choose this period because we would like to make our numbers broadly comparable to the aggregate balance sheet data of FDIC-insured commercial banks and savings institutions. The FDIC made the balance sheet data for this period available, and both the mortgage rate and federal funds rate data are obtained from Table H.15 published by the Federal Reserve.

The income from deposit services is important in bank liability structure. The net income from deposit services should be determined in a competitive market for deposits. In a perfect competitive market with free entry, the net income would be driven to zero, or it should just cover the insurance premium if a bank has deposit insurance. At least in the U.S., new entry of

banks into the market is regulated by charter authorities. Founders of a new bank have to show their integrity and ability to manage the bank. In addition, the regulators demand evidence of need for a new bank before granting a charter. Peltzman (1965) documents the restriction on entry of commercial banking. Jayaratne and Strahan (1998) examines the effects of entry restrictions on bank efficiency.

Without free entry, deposit rents arise from the market power enjoyed by the bank (De Nicolo and Rurk Ariss, 2010). The profitability should depend on the amount of deposits and the bank. Thus, parameter  $\eta$  may differ across banks and should be a function of  $D$ . We do not explicitly model the market equilibrium of deposits or the demand function of deposits,  $D(\eta)$ , in order to keep the model tractable and to focus on the choice of liability structure. We instead assume  $\eta$  to be a constant but allow it to be different across banks. A range of values,  $\eta \in [1\%, 3.5\%]$ , are examined in our analysis of comparative statics.

Since a benefit of leverage is the tax deductibility of interest expenses, corporate tax rate is an important parameter in determining liability structure. The statutory corporate tax rate in the U.S. ranges up to 35%. The U.S. Department of Treasury (2007) reports that the effective marginal tax rate on investment in business varies substantially by business sectors. The academic literature suggests that the effective corporate tax rate is around 10% for non-financial firms (Graham, 2000) but can be more important for banks (Heckemeyer and Mooij, 2013). In our analysis of comparative statics, we consider a wide range,  $\tau \in [2\%, 20\%]$ .

Bankruptcy cost is an important countering force to the benefit of leverage, but the task of measuring the cost has always been a challenge. A well-known reference is the study of Altman (1984), which examines a sample of 19 industrial firms that went bankrupt over the period of 1970–1978. The estimated bankruptcy cost is 19.7% of the firm’s asset value just prior to its bankruptcy. Bris, Welch and Zhu (2006), however, show that bankruptcy cost varies across firms and ranges between 0% and 20% of firm assets. Banks experienced higher bankruptcy costs. Based on 791 FDIC-regulated commercial banks failed during 1982–1988 (the Savings and Loan Crisis), James (1991) estimates that bankruptcy cost is 30% of a failed bank’s assets. Based on 325 insured depository institutions failed during 2008–2010 (the Great Recession), Flannary (2011) estimates that bankruptcy cost is about 27% of a failed bank’s assets. In light of these estimates, we choose 27% as the mid-point of the range. The range we consider is  $[15\%, 40\%]$ .

The other exogenous parameters are as follows. A state banking regulatory agency closes a bank when it is unable to meet its obligations to depositors. The parameter  $\beta$  should not be lower than 0. When a bank’s total capital is less than 2% of its assets, the FDIC classifies it as “critically undercapitalized,” and the charter authority typically closes the bank. In view of these institutional arrangements, we set the regulatory closure rule as  $\beta = 0.02$ . For insurance subsidy  $\omega$ , we examine a wide range, which is from zero to 40%.

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