Shadow Banking and Financial Stability under Limited Deposit Insurance

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Abstract

I study the relation between shadow banking and financial stability in an economy in which banks are susceptible to self-fulfilling runs and in which government-backed deposit insurance is limited. Shadow banks issue only uninsured deposits while commercial banks issue both insured and uninsured deposits. The effect of shadow banking on financial stability is ambiguous and depends on the (exogenous) upper limit on insured deposits. If the upper limit on insured deposits is high, then the presence of a shadow banking sector is detrimental to financial stability; shadow banking creates systemic instability that would not be present if all deposits were held in the commercial banking sector. In contrast, if the upper limit on insured deposits is low, then the presence of a shadow banking sector is beneficial from a financial stability perspective; shadow banks absorb uninsured (and uninsurable) deposits from the commercial banking sector, thereby shielding commercial banks from runs. While runs may occur in the shadow banking sector, the situation without shadow banks and a larger amount of uninsured deposits held at commercial banks is worse.

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1 Introduction

The recent decades have witnessed the growth of a so-called shadow banking sector in the United States, which provides very short-term claims similar to bank deposits outside the traditional banking system (Poszar et al. 2010 and Ricks 2012 provide good overviews).[1] Prominent examples of shadow bank claims are money market mutual fund shares, overnight asset backed commercial paper, or certain forms of repo.[2] Since the financial crisis of 2007-08, and especially since the run on money market mutual funds in September 2008[3], the shadow banking sector is widely thought to pose a threat to financial stability.

In this paper, I present a theoretical argument why the financial stability implications of the shadow banking sector should not be analyzed separately from the cap on deposit insurance at traditional banks. Shadow banks cater mostly to institutional investors managing large cash-balances, who have a preference for extremely safe, short-term assets (Poszar 2011). For instance, cash pools of large non-financial corporations today commonly amount to several hundred million USD, a large part of which is held in the form of shadow bank liabilities rather than traditional bank deposits (Poszar 2011). Given the cap on deposit insurance, it is impossible or impracticable for these institutional cash pools to hold all of their funds in the form of insured bank deposits. In this context of limited deposit insurance, shadow banks can have the effect of absorbing uninsured (and uninsurable) short-term claims from the commercial banking sector. I show that this aspect of shadow banking may be beneficial from a financial stability perspective and a flow of uninsured deposits back into the commercial banking sector can be detrimental to aggregate financial stability.

[1] There is no universally accepted definition of the term shadow banking. Some understand the term broadly, encompassing various sorts of financial intermediation outside the traditional banking system (e.g. FSB (2017)). Others define shadow banking more narrowly as the provision of 'money-like' (or bank-deposit-like) liabilities outside the traditional banking system (e.g. Poszar (2014)). This paper has the latter, narrow definition in mind.

[2] Short-term asset backed commercial paper has diminished in importance since the financial crisis. The money market mutual fund industry is in flux after reforms enacted in 2016 (see Cipriani et al. (2017)).

[3] Schmidt et al. (2016) provide a detailed description of the run on money market mutual funds in 2008. Episodes that can be characterized as bank runs were also observed in other segments of the shadow banking system such as the market for short-term asset backed commercial paper (Covitz et al. 2013, Kacperczyk and Schnabl 2010).

I take the deposit insurance scheme, and in particular the cap on deposit insurance, as exogenous throughout the paper. Deriving the optimal level of the cap is not the subject of this paper. For a given deposit insurance scheme in place, I derive the structure of the financial system for which aggregate losses caused by systemic bank runs are minimized, and I compare it to the structure of the financial system that results in a competitive equilibrium. In this sense, the paper speaks to a regulator that cannot change the deposit insurance scheme in place.

Model Summary The model features banks that sell claims which are redeemable on demand ('deposits') to households. Banks invest into riskless projects whose maturity exceeds the maturity of deposits. The short-term nature of the claims issued by banks is taken as given. Households choose at which banks to hold their deposits. In addition, households can choose to obtain deposit insurance for a limited amount of deposits. The limit on insured deposits is given by an exogenous parameter representing the cap on deposit insurance. The cap on deposit insurance amounts to a rationing of deposit insurance; it implies that some fraction of deposits are 'uninsurable'. The lower the deposit insurance cap, the higher the amount of uninsurable deposits. The only dimension in which banks differ from each other is the share of insured and uninsured deposits among the deposits issued. If all deposits issued by a bank are uninsured, the bank is labelled a 'shadow bank'. If some (not necessarily all) of the deposits issued by a bank are insured by deposit insurance, the bank is labelled a 'commercial bank'.[4]

Since banks engage in maturity transformation, they are illiquid in an interim period, which opens up the possibility of self-fulfilling bank runs in the spirit of Diamond and Dybvig (1983). Depositors never have an incentive to run on their insured deposits, which means that only uninsured deposits may potentially be withdrawn in a run. If a bank is hit by a run, it needs to sell assets on a secondary market. The liquidation price of assets in the secondary market is decreasing in the total amount of assets liquidated by banks ('cash-in-the-market pricing'). An individual bank is susceptible to a run if (i) the share of uninsured deposits at the bank is high and (ii) the liquidation

[4]In order to focus on the aspects most relevant for the theme of this paper, I abstract from other differences between shadow- and commercial banks. In reality, shadow- and commercial banks differ in more dimensions than just the fact that shadow bank liabilities are not explicitly protected by government-sponsored deposit insurance.

price of assets is low. The liquidation price itself depends on how many other banks are hit by a run and sell assets, which introduces a systemic element to bank runs. The model abstracts from fundamental risk, so that these self-fulfilling, systemic bank runs constitute the only source of risk in the economy.

Main Results First I show that, for a given total amount of insured and uninsured deposits outstanding in the economy, the set of banks that can potentially be affected by a systemic run depends on the distribution of insured and uninsured deposits across banks. In general, the magnitude of systemic bank runs is minimized if the financial system exhibits a dual structure, with one sector that issues both insured and uninsured deposits (the commercial banking sector) and another sector that issues only uninsured deposits (the shadow banking sector). While systemic runs may occur in the shadow banking sector, the presence of the shadow banking sector also implies that the share of insured deposits at commercial banks is relatively high, so that systemic runs do not encompass the commercial banking sector. For this reason, a shadow banking sector can be beneficial from a financial stability perspective if the deposit insurance cap is low, that is, if the share of uninsurable deposits in the economy is high. To minimize aggregate losses caused by runs, the shadow banking sector should be set to the smallest size at which it absorbs enough of the uninsurable deposits from the commercial banking sector such as to keep the commercial banking sector shielded from systemic runs.

Next, I analyze the structure of the financial system that results in a competitive equilibrium and compare it to the optimal allocation as described above. Households may face conflicting incentives regarding the type of bank at which they hold deposits. On the one hand, the presence of insured depositors who do not participate in runs reduces expected losses caused by runs for uninsured depositors at commercial banks. This gives households an incentive to hold uninsured (and uninsurable) deposits at commercial banks rather than shadow banks. On the other hand, if the deposit insurance agency charges a fee on deposits issued by commercial banks, households have an incentive to move into shadow banks in order to avoid the fee. The shadow banking sector tends to be smaller than optimal if the share of uninsurable deposits is high (that is, if the deposit insurance

cap is low). Intuitively, if aggregate financial stability is low, households have an incentive to move uninsurable deposits from the shadow banking sector into the more stable commercial banking sector, thereby increasing the share of uninsured deposits held at commercial banks, which causes the commercial banking sector to become susceptible to runs as well. In contrast, the shadow banking sector tends to be larger than optimal if the share of uninsurable deposits is low (that is, if the deposit insurance cap is high). In this case, the commercial banking sector will not be susceptible to systemic runs even if most (or all) uninsurable deposits are held at commercial banks. In a competitive equilibrium, households move deposits into the shadow banking sector in order to avoid the fee on commercial bank deposits. The equilibrium size of the shadow banking sector is such that shadow banks are susceptible to systemic runs and households are indifferent at the margin between investing in shadow banks prone to runs and paying the fee on commercial bank deposits.

A regulator aiming to implement the optimal size of the shadow banking sector, taking as given the deposit insurance cap in place, can achieve this with a two-pronged policy. First, impose a tax on shadow bank deposits that mimics the fee charged on commercial bank deposits. This prevents the shadow banking sector from growing too large relative to the optimal size. Second, impose a marginal tax on uninsured commercial bank deposits that exceed a certain amount. This limits the amount of uninsured deposits held at commercial banks and ensures that the shadow banking sector is not too small relative to the optimal size.

Related Literature In a recent paper, Davila and Goldstein (2016) study the optimal level of the cap on deposit insurance, including the case where runs have a systemic element as in the present paper. Increasing the cap has the benefit of reducing expected losses caused by runs but entails social costs such as deadweight losses of taxation. The present paper is complementary to Davila and Goldstein (2016) by showing that the trade-offs studied in Davila and Goldstein (2016) can be improved if, in addition to choosing the level of the cap, an appropriate distribution of insured and uninsured deposits across banks can be implemented. Another closely related paper is Luck and Schempp (2016) who study financial stability implications of the shadow banking sector in an economy in which commercial banks issue insured- and shadow banks uninsured short-term claims.

The magnitude of systemic runs increases in the size of the shadow banking sector. The present paper shows that some of the conclusions reached in Luck and Schempp (2016) regarding shadow banking and financial stability may be reversed if deposit insurance is limited and commercial banks issue both insured and uninsured deposits.

More generally, this paper is related to a recent theoretical literature studying the financial stability implications of the shadow banking sector. Hanson et al. (2015) characterize shadow banking and commercial banking as two different ways to provide riskless claims. Shadow banks create riskless claims by investing in relatively liquid assets that can be liquidated immediately if bad news arrive. In this sense the occurrence of fire sales in the shadow banking sector is inherent to shadow banks' business model. In Gertler et al. (2016) shadow banks are modelled as wholesale banks that issue debt to other (retail) banks. Due to a relatively low degree of agency frictions compared to retail banking, shadow banking can reduce the financial accelerator in the aftermath of real shocks. However, high leverage in the shadow banking sector can also lead to instability in the form of bank runs. Moreira and Savov (2017) characterize shadow banking as the provision of risky claims which are information-insensitive and therefore provide liquidity services. This leads to a socially desirable expansion of liquidity in normal times but makes the economy more vulnerable to changes in aggregate uncertainty. Martin et al. (2014) study run equilibria on various types of shadow banks, taking into account the specifics of the debt contracts used. Gennaioli et al. (2013) focus on shadow banks' role in the securitization process. While securitization allows for gains from trade between risk averse buyers of securities and risk neutral financial intermediaries, it also makes the financial system more vulnerable to aggregate shocks. Compared to the papers mentioned above, the present paper highlights that, in the context of limited deposit insurance, the presence or absence of a shadow banking sector affects the distribution of uninsurable short-term claims across different financial institutions, with consequences for financial stability. This paper abstracts of many issues relevant to shadow banking and should be seen as complementary to the papers mentioned above.

Finally, this paper's interest in the effect of deposit insurance design on the equilibrium structure of a financial system populated (potentially) by both commercial banks and shadow banks is shared by two recent papers by LeRoy and Singhania (2017) and Chrétien and Lyonnet (2017), albeit with a somewhat different focus. LeRoy and Singhania (2017) study how deposit insurance pricing affects equilibrium portfolio choices of commercial banks and shadow banks. In Chrétien and Lyonnet (2017), the deposit insurance scheme allows commercial banks to issue riskless debt in times of crisis, which enables them to act as a 'buyer of last resort' for assets usually held by shadow banks. This leads to a complementarity between commercial banking and shadow banking, and an extension of the deposit insurance scheme for commercial banks indirectly benefits shadow banks as well. In order to focus on the aspects that are most relevant for the main theme of this paper, I abstract from banks' portfolio choices and treat the secondary market for banks' assets as exogenous.

The remainder of the paper is structured as follows. Section 2 describes the environment. Section 3 discusses run equilibria. Section 4 derives the optimal structure of the financial system. Sections 5 and 6 discuss the competitive equilibria of the economy, first for the case where deposit insurance is costless for households (section 5) and then for the case where a fee is charged on commercial bank deposits (section 6). Section 7 discusses how the optimal structure of the financial system can be implemented in a competitive equilibrium.

2 The Model

The economy lasts for two periods, indexed by t=0,1. Period 1 is subdivided into beginning of period and end of period. There is an infinitely divisible good used for consumption and investment. Two types of agents populate the economy in period 0: A continuum of households, indexed by $h \in \mathcal{H} = [0, 1]$ and a continuum of banks, indexed by $i \in \mathcal{I} = [0, 1]$. Each household is born with an endowment of one unit of good. Banks are born without endowment. At the beginning of

period 1, a continuum $[0, 1]$ of agents called 'outside investors' are born, each with an endowment of $\lambda^S \in (0, 1)$ units of good.

In period 0, there is a riskless, constant returns to scale investment technology that returns one unit of good at the end of period 1 per unit of good invested in period 0. Investments cannot be terminated prematurely at the beginning of period 1. There is no other storage technology between periods 0 and 1.

Households maximize expected utility $E[u(c_1)]$, where $c_1 \geqslant 0$ is defined as total consumption during period 1. Households are therefore indifferent about whether to consume at the beginning or at the end of period 1. Utility $u(\cdot)$ is strictly increasing, strictly concave and twice continuously differentiable. Banks' utility is strictly increasing in both period 0 and period 1 consumption. Outside investors' utility is strictly increasing in period 1 consumption.

For reasons that are outside of the model, households only save in the form of demand deposits, which can be issued by banks in period 0. Banks can invest the proceeds from the sale of demand deposits in period 0 into the investment technology. Demand deposits issued by any bank i:

(i) stipulate the return $r(i)$ which the bank pays to depositors at the end of period 1, per unit invested into the bank in period 0.[5]

(ii) allow depositors to withdraw an amount of good equal to the face value of the deposit already at the beginning of period 1. Depositors who withdraw early are served sequentially in random order, as in Diamond and Dybvig (1983).

Since the investment technology pays out only at the end of period 1, banks are illiquid at the beginning of period 1. If households withdraw early, banks need to raise good by selling claims to the investment return to outside investors. Note that, since investments are fundamentally riskless, losses incurred by banks are always related to liquidation losses incurred at the beginning of period 1. Households are indifferent about when to consume during period 1, and I assume they withdraw early only if they have a strict incentive to do so.

[5]Competition among banks is going to imply that $r(i) = 1$ for all banks.

The final element of the model is an exogenous scheme of limited deposit insurance. In period 0, after having bought demand deposits from banks, households can choose for which deposits to obtain deposit insurance. If households obtain deposit insurance for some of the deposits they bought, they are guaranteed to receive an amount of good equal to the face value of the deposits at the end of period 1. Whenever a bank is not able to pay out an amount of good corresponding to the face value of the insured deposits at the end of period 1, deposit insurance makes up for the difference. The total face value of insured deposits held by a household is limited to $\theta \in [0, 1]$, where θ represents the 'cap' on deposit insurance. The cap amounts to a rationing of deposit insurance. Different to real-world deposit insurance arrangements, the cap is a cap per person, without a specific limit on insured deposits held at a certain bank.[6] Depositors may choose to obtain deposit insurance for some, but not all, of the deposits held at one particular bank. As a result, among the deposits issued by a given bank, some may be insured by deposit insurance while others are not.

In the baseline version of the model, households can obtain deposit insurance for deposits issued by all banks at no cost.[7] Deposit insurance payments are financed by levying a lump-sum tax on all households at the end of period 1.[8] The deposit insurance agency remains passive as events unfold. If banks have both insured and uninsured deposits outstanding at the end of period 1, banks are allowed to pay their uninsured depositors first, thereby shifting losses (in case the bank incurred losses) towards deposit insurance.[9] I abstract from any further issues related to moral hazard or asymmetric information; the face value of deposits sold in period 0 must be backed by a corresponding amount of real investment and banks commit to pay depositors in period 1 whenever they can, rather than running away with the investment return. Figure 1 sketches the timeline.

[6] Modelling the cap as a cap per person and bank would call for a richer model that endogenizes the number of banks that offer insured deposits in equilibrium, for instance by introducing a fixed cost of opening a bank. See also section 6.

[7] In section 6, I discuss a version of the model where banks can choose whether or not get access to deposit insurance, and a fee is charged on all deposits issued by banks with access to deposit insurance.

[8] Consumption c_1 equals the total return received during period 1 from a household's investment into deposits (including any payment by deposit insurance in case some banks failed) minus taxes to deposit insurance. Since deposit insurance payments represent transfers from households to themselves, all households can pay the lump-sum tax in a symmetric allocation. In a hypothetical non-symmetric allocation in which some households' consumption level c_1 would go to negative if they paid the entire tax, these households consume $c_1 = 0$ and the tax will be increased accordingly for the remaining households.

[9] Without loss of generality, I assume banks always make use of this possibility. See Schilling (2018) for a setting where the deposit insurance agency acts strategically and sets its forbearance policy optimally for given levels of deposit insurance coverage.

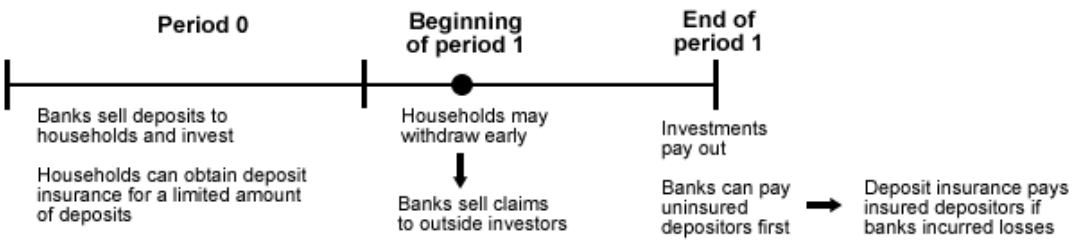

Figure 1: Timeline.

3 Runs

When faced with withdrawals at the beginning of period 1, banks need to sell assets (claims to investment return) to outside investors in order to pay out the depositors who withdraw.[10] As in Diamond and Dybvig (1983), orders of withdrawals are processed sequentially in random order and depositors are paid out at face value.[11] Let p denote the price, at the beginning of period 1, of an asset that pays out one unit of good at the end of period 1. Since there is no discounting within period 1, outside investors will buy assets at a price of $p = 1$ (the fundamental value) as long as their endowment is sufficient to do so. Let λ^D denote the total fundamental value of all assets sold at the beginning of period 1. If $\lambda^D > \lambda^S$ then outside investors' aggregate endowment is not sufficient to buy all assets sold in the beginning of period 1 at their fundamental value, and the market-clearing price is determined by cash-in-the-market pricing à-la Allen and Gale (1994):

$$\underbrace{p(\lambda^D)}_{\text{liquidation price}} = \min\left\{\frac{\overbrace{\lambda^S}^{\text{secondary market capacity}}}{\underbrace{\lambda^D}_{\text{assets sold}}}, 1\right\} \tag{1}$$

From Diamond and Dybvig (1983) we know that the combination of payment-on-demand deposits and liquidation losses can lead to self-fulfilling run equilibria. Liquidation losses occur whenever assets trade below fundamental value. However, since depositors never have an incentive to with-

[10] A 'depositor' is a household that holds deposits (insured or uninsured) at a bank.

[11] The order of the line is determined independently at each bank, which allows households to diversify away idiosyncratic risk regarding the order in the line by spreading their investment over many banks. See also the discussion in section 5 and footnote 20.

draw insured deposits early, susceptibility to runs depends also on the share of insured deposits among the deposits issued by a bank.

Throughout this section, I assume that the total face value of deposits issued by a bank corresponds to the fundamental value of assets held by the bank.[12] Denote $\vartheta(i)$ as the share of insured deposits (in terms of the face value) among all deposits issued by bank i.[13] To illustrate how susceptibility to runs depends both on the share of insured deposits $\vartheta(i)$ and on the liquidation price p, consider a bank with 50% insured deposits ($\vartheta(i) = 0.5$) and suppose the liquidation price at the beginning of period 1 satisfies $p < 0.5$. Then the bank cannot pay out all uninsured depositors if they all withdraw at the beginning of period 1, even by liquidating the entire portfolio at the current market price $p < 0.5$. The bank is then susceptible to self-fulfilling runs since nothing will be left in the bank for the uninsured depositor that shows up last at the bank, in case all other uninsured depositors withdraw. Suppose now the liquidation price equals $p = 0.8$. The bank then could pay out all uninsured depositors if they all withdraw at the beginning of period 1 by selling a fraction $\frac{0.5}{0.8} = 0.625$ of its portfolio at the current market price $p = 0.8$. No matter how many uninsured depositors withdraw early, the bank will always have funds left at the end of period 1 to pay out the remaining uninsured depositors.[14] It follows that uninsured depositors have no incentive to participate in a run at the beginning of period 1, so that the bank is not susceptible to runs. In general, and by the same reasoning, a bank will be susceptible to runs if and only if:

$$\underbrace{1 - \vartheta(i)}_{\substack{\text{share of}\\ \text{uninsured}\\ \text{deposits}}} > \underbrace{p}_{\substack{\text{liquidation}\\ \text{price}}} \tag{2}$$

A bank with only insured depositors ($\vartheta(i) = 1$) will never be susceptible to runs, independent of the liquidation price. A bank with no insured depositors ($\vartheta(i) = 0$) will be susceptible to a

[12] In principle, the total face value of deposits issued by a bank could be lower than the fundamental value of the assets held by the bank. Since competition drives down bank profits to zero, this will not occur in equilibrium. (See section 5.)

[13] If a bank does not issue any deposits, then $\vartheta(i) = 0$.

[14] The assumption that the deposit insurance agency remains entirely passive as events unfold is important here. The deposit insurance agency allows banks (i) to liquidate assets at a loss at the beginning of period 1 in order to pay out uninsured depositors and (ii) to pay out uninsured depositors first at the end of period 1, thereby shifting losses to deposit insurance.

run whenever $p < 1$, that is, whenever assets trade below fundamental value. Banks that are hit by a run sell their entire portfolio on the secondary market. By cash-in-the-market pricing (1), the liquidation price p thus depends on how many banks are hit by a run. This introduces a systemic element to runs; the larger the number of banks hit by a run, the larger the number of banks susceptible to a run.

Denote $D(\mathcal{I}')$ as the total fundamental value of assets held by some subset of banks $\mathcal{I}' \subseteq \mathcal{I}$, which is identical to the total face value of deposits issued by banks $\mathcal{I}'$. The liquidation price on the secondary market in a scenario where all banks in $\mathcal{I}'$ are hit by a run then equals $p(D(\mathcal{I}'))$. The economy is said to exhibit a *run equilibrium encompassing all banks in* $\mathcal{I}'$ iff:[15]

$$1 - \vartheta(i) > p(D(\mathcal{I}')) \qquad \text{for all banks} \quad i \in \mathcal{I}' \tag{3}$$

If condition (3) is fulfilled then, given that all banks in $\mathcal{I}'$ are hit by a run, all banks in $\mathcal{I}'$ are susceptible to a run. We proceed with the following result:

Lemma 3.1. *Let $f(\vartheta) = 1 - p\big(D(i \,|\, \vartheta(i) \leqslant \vartheta)\big)$. Then $f(\vartheta)$ has a greatest fixed point, denoted by ϑ^{sr}. Furthermore:*

(i) The economy exhibits a run equilibrium encompassing all banks with $\vartheta(i) < \vartheta^{sr}$

(ii) The economy does not exhibit a run equilibrium encompassing banks with $\vartheta(i) \geqslant \vartheta^{sr}$

Proof: By (1), the liquidation price p is decreasing in the number of banks that liquidate their portfolio. It follows that $f(\vartheta)$ is an increasing function mapping $[0, 1]$ into itself. By Tarski's fixed point theorem, the set of fixed points of $f(\vartheta)$ is non-empty and has a greatest element, which will be denoted by ϑ^{sr}. We also have that $\vartheta \geqslant f(\vartheta)$ for any $\vartheta \geqslant \vartheta^{sr}$. By (3), there exists a run equilibrium encompassing all banks with $\vartheta(i) < \vartheta^{sr}$ if:

$$1 - \vartheta(i) > p(D(i \,|\, \vartheta(i) < \vartheta^{sr})) \qquad \text{for any} \quad \vartheta(i) < \vartheta^{sr} \tag{4}$$

[15] According to this definition, a run may encompass banks that do not have any depositors, which by definition have $\vartheta(i) = 0$. A "run" that only encompasses banks without depositors is not possible however. If no assets are sold on the secondary market, then $p = 1$ which means that banks with $\vartheta(i) = 0$ are not susceptible to a run.

Rewriting condition 4 yields:

$$\vartheta(i) < 1-p(D(i\,|\,\vartheta(i) < \vartheta^{sr})) \leqslant 1-p(D(i\,|\,\vartheta(i) \leqslant \vartheta^{sr})) = f(\vartheta^{sr}) = \vartheta^{sr} \quad \text{for any} \quad \vartheta(i) < \vartheta^{sr}$$

Hence condition 4 is fulfilled and the economy exhibits a run equilibrium encompassing all banks with $\vartheta(i) < \vartheta^{sr}$. Suppose next that, in contradiction to item (ii) in lemma 3.1, the economy exhibits a run equilibrium encompassing a bank with $\vartheta(i) = \tilde{\vartheta} \geqslant \vartheta^{sr}$. Then the economy exhibits a run equilibrium encompassing all banks with $\vartheta(i) \leqslant \tilde{\vartheta}$. (This follows from the fact that, by (2), any bank that is susceptible to a run at some liquidation price p' will also be susceptible to run at any liquidation price $p'' \leqslant p'$). Repeating the same steps as before, the economy exhibits a run equilibrium encompassing all banks with $\vartheta(i) \leqslant \tilde{\vartheta}$ only if:

$$\vartheta(i) < 1 - p(D(i\,|\vartheta(i) \leqslant \tilde{\vartheta})) = f(\tilde{\vartheta}) \quad \text{for any} \quad \vartheta(i) \leqslant \tilde{\vartheta} \tag{5}$$

Since $\tilde{\vartheta} \geqslant f(\tilde{\vartheta})$, condition 5 is violated and we arrive at a contradiction. ■

The main implication of lemma 3.1 is that a run encompassing all banks with $\vartheta(i) < \vartheta^{sr}$ is the largest run that is possible, that is, the run encompassing the largest set of banks. In the remainder of the paper, a *systemic run* denotes a run encompassing all banks with $\vartheta(i) < \vartheta^{sr}$. The total fundamental value of assets liquidated in a systemic run is given by $D\left(i \in \mathcal{I}\,|\,\vartheta(i) < \vartheta^{sr}\right)$ and will sometimes be referred to as the *magnitude* of systemic runs. For future reference it will be useful to denote p^{sr} as the liquidation price in a systemic run:

$$p^{sr} = \min\left\{\frac{\overbrace{\lambda^S}^{\substack{\text{secondary}\\\text{market}\\\text{capacity}}}}{\underbrace{D\left(i\,|\,\vartheta(i) < \vartheta^{sr}\right)}_{\substack{\text{total assets sold}\\\text{in systemic run}}}}, 1\right\} \tag{6}$$

Note that, if the fixed point in lemma 3.1 is given by $\vartheta^{sr} = 0$, then the economy does not exhibit a run equilibrium, and we have $p^{sr} = 1$. It will also be useful to denote $\varphi(\vartheta(i), p^{sr})$ as the fraction

of uninsured deposits that bank i can pay out in case of a systemic run:

$$\varphi(\vartheta(i), p^{sr}) = \min\left\{\frac{\overbrace{p^{sr}}^{\text{liquidation price in run}}}{\underbrace{1-\vartheta(i)}_{\text{share of uninsured deposits}}}, 1\right\} \tag{7}$$

The fraction of uninsured deposits a bank can pay out in a systemic run increases in the share of insured deposits at the bank, $\vartheta(i)$, and the liquidation price p^{sr}. If bank i is not susceptible to runs, then we have $\varphi(\cdot) = 1$.

Equilibrium selection is driven by an exogenous sunspot variable $\xi \in \{0, 1\}$ that realizes at the beginning of period 1. Households select the no-run equilibrium if $\xi = 0$ occurs and they select the systemic run equilibrium if $\xi = 1$ occurs, in case the economy exhibits a systemic run equilibrium.[16] $\xi = 1$ occurs with some probability $\pi^r > 0$ and $\xi = 0$ occurs with probability $1 - \pi^r$.

I will now illustrate by way of an example why the magnitude of systemic runs in this economy depends on the distribution of insured and uninsured deposits across banks. Figure 2 shows three alternative structures of the financial system, with an identical total amount of insured deposits (in grey) and uninsured deposits (in white) outstanding. Secondary market capacity is given by $\lambda^S = 0.25$.

Figure 2: Three alternative distributions of insured and uninsured deposits across banks.

[16] Households select either the no-run equilibrium or the systemic run equilibrium. In principle the economy may exhibit many different run equilibria in which smaller subsets of banks are hit by a run.

The left-hand side of figure 2 shows a financial system in which insured and uninsured deposits are distributed uniformly across banks. There is one representative bank labelled A, which may stand for many identical banks, with 50% insured deposits. Consider a hypothetical situation in which all banks A liquidate their portfolios. The liquidation price then falls to $p = \frac{0.25}{1} = 0.25$. Since $0.25 < 0.5$, all banks A are susceptible to runs at a liquidation price of $p = 0.25$ (see condition 2). It follows that systemic runs in this economy encompass the entire financial system.

The middle of figure 2 shows a dual financial system in which deposits are distributed in such a way that all insured deposits are held in sector A (the 'commercial banking sector') of the financial system and all uninsured deposits are held in sector B (the 'shadow banking sector'). Banks A are never susceptible to runs. Consider now a hypothetical situation in which all banks B liquidate their portfolios. The liquidation price then falls to $p = \frac{0.25}{0.5} = 0.5$. Since $0.5 < 1$, all banks B are susceptible to runs at this liquidation price. It follows that there is a systemic run equilibrium that encompasses the entire sector B of the financial system. Compared to the uniform structure depicted on the left-hand side of figure 2, systemic runs encompass only half of the financial system.

The extreme distribution of insured and uninsured deposits across banks that is depicted in the middle of figure 2 does not in general minimize the magnitude of systemic runs. To see this suppose that, starting from the situation depicted in the middle of figure 2, a certain amount of uninsured deposits is moved from sector B into sector A. This leads to the situation depicted on the right-hand side of figure 2. The share of uninsured deposits in sector A now equals $\frac{0.1}{0.6} < 0.25$. Since there is no scenario in which the liquidation price falls below 0.25, there is still no systemic run equilibrium that encompasses sector A of the financial system. Sector B, which is still susceptible to systemic runs, is smaller compared to the previous situation. As a result, a relatively smaller part of the financial system is susceptible to systemic runs compared to the situation depicted in the middle of figure 2. In general, the magnitude of systemic runs is minimized by setting sector B to the smallest size at which it is large enough to absorb enough of the uninsurable deposits from sector A to make sure that sector A is not susceptible to systemic runs.

4 Optimal Structure of the Financial System

In this section, I derive the structure of the financial system that maximizes welfare, taken as given the deposit insurance cap θ. Welfare is defined as the integral over expected utility of households, with equal weight given to all households. The structure of the financial system is given by two integrable functions $(\delta(i), \vartheta(i))$, where $\delta(i) : \mathcal{I} \to \mathbb{R}^+$ denotes the face value of deposits issued by bank i and, as before, $\vartheta(i) : \mathcal{I} \to [0, 1]$ denotes the share of insured deposits among the deposits issued by bank i.

I impose two additional restrictions on the welfare maximization problem. First, all households must receive an identical portfolio of deposits in period 0. This rules out the (arguably uninteresting) allocation where each bank serves exactly one household, which would eliminate the coordination problem inherent to bank runs. Second, the aggregate face value of deposits must be equal to 1, which implies that the entire period 0-endowment is invested, and the aggregate face value of deposits equals the aggregate investment return. This is equivalent to saying that the aggregate face value of deposits must be the same as in a competitive allocation (see section 5). This reflects the fact that the present paper is concerned with the optimal distribution of insured and uninsured deposits across banks, not with the optimal amount of short-term debt. It also implies that the total face value of deposits issued by some set $\mathcal{I}'$ of banks must be identical (not lower) than the fundamental value of assets held by these banks. As before $D(\mathcal{I}') = \int_{\mathcal{I}'} \delta(i)\, di$ denotes the total face value of deposits issued by banks $\mathcal{I}'$, which corresponds to the total fundamental value of assets held by banks $\mathcal{I}'$.

The only uncertainty at the aggregate level stems from the realization of the sunspot variable ξ. By a law of large numbers, idiosyncratic risk regarding the order in the line at individual banks in case of a systemic run is eliminated, given that each depositor holds deposits at a continuum of different banks.[17] Consumption c_1 of depositors is thus only subject to aggregate risk related

[17] There are well known technical problems with the law of large numbers in economies with a continuum of agents - see also footnote 20.

to the realization of ξ. If households select the no-run equilibrium ($\xi = 0$), then consumption of households in period 1 is equal to the fundamental value (the end of period 1 return) of the period 0 investment. If households select the systemic run equilibrium ($\xi = 1$), then consumption of households equals the fundamental value of the period 0 investment minus losses caused by the run. Total losses in a systemic run (from the point of households) are equal to the fundamental value of claims sold to outside investors minus the amount outside investors pay for it. The latter simply equals outside investors' total endowment λ^S. Recall that a systemic run encompasses all banks whose share of insured deposits is strictly below ϑ^{sr}, where ϑ^{sr} is the fixed point defined in lemma 3.1. Furthermore, any payments made by deposit insurance in period 1 represent transfers from households to themselves, so that they cancel out in the aggregate. Consumption of households in case of no run ($\xi = 0$) and a systemic run ($\xi = 1$) respectively can thus be expressed as:

$$
\begin{aligned}
c(0) &= 1 \\
c(1) &= 1 - \max\Big\{ \underbrace{\overbrace{D(i \in \mathcal{I} | \vartheta(i) < \vartheta^{sr})}^{\text{fundamental value of assets sold in systemic run}} - \overbrace{\lambda^S}^{\text{secondary market capacity}}}_{\text{total loss caused by systemic run}}, 0 \Big\}
\end{aligned}
\tag{8}
$$

Note that if systemic runs cannot occur in the economy, then $D(i \,|\, \vartheta(i) < \vartheta^{sr}) = 0$ and we have $c(1) = 1$. The optimal structure of the financial system is defined as any $(\delta(i), \vartheta(i))$ solving:

$$
\max_{(\delta(i),\vartheta(i))} \overbrace{E\big[u(c_1(\xi))\big]}^{\text{expected utility of households}} \quad \text{subject to} \quad \overbrace{\int_{\mathcal{I}} \delta(i)\, di}^{\text{total face value of deposits}} = 1 \quad \text{and} \quad \overbrace{\int_{\mathcal{I}} \vartheta(i)\, \delta(i)\, di}^{\text{face value of insured deposits}} \leqslant \theta \tag{9}
$$

It follows immediately from (8) and (9) that the optimal structure of the financial system is such that it minimizes the total loss caused by a systemic run. We proceed with the following result:

Lemma 4.1. *The optimal structure of the financial system satisfies*

$$
\underbrace{D(i \in \mathcal{I} \,|\, \vartheta(i) = 0)}_{\substack{\textit{face value of deposits issued} \\ \textit{by banks with } \vartheta(i) = 0}} + \underbrace{D(i \in \mathcal{I} \,|\, \vartheta(i) \geqslant \vartheta^{sr})}_{\substack{\textit{face value of deposits issued} \\ \textit{by banks with } \vartheta(i) \geqslant \vartheta^{sr}}} = 1.
$$

Stated verbally, lemma 4.1 says that insured and uninsured deposits are distributed across banks in such a way that there are at most two types of banks: banks with no insured deposits ($\vartheta(i) = 0$) and banks with enough insured deposits to prevent them from being susceptible to systemic runs ($\vartheta(i) \geqslant \vartheta^{sr}$). Figure 3 provides a graphical illustration of the proof of lemma 4.1. Suppose there is a set of banks with $\vartheta \in (0, \vartheta^{sr})$ that issue some amount $D > 0$ of deposits. The fundamental value of assets sold by these banks in a systemic run equals D. Suppose now one reallocates an amount D' of uninsured deposits away from these banks into new banks, where D' is such that the share of insured deposits at the original banks reaches ϑ^{sr}. This reallocation of deposits is illustrated in figure 3. An amount $D - D'$ of deposits are now held at banks that are run-proof, which decreases the amount of assets liquidated in a systemic run and lowers the total loss caused by a systemic runs. Hence the initial structure of the financial system cannot be optimal.

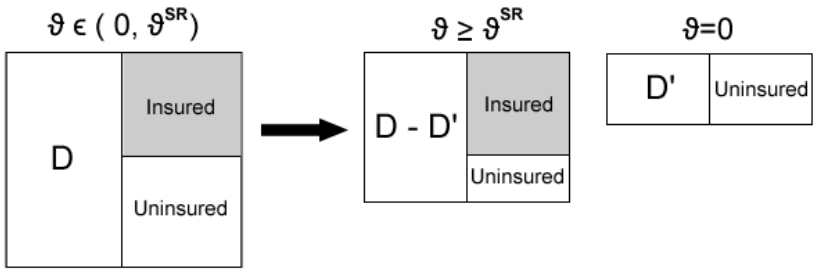

Figure 3: Illustration of the proof of lemma 4.1

In the following, I will label banks with no insured deposits ($\vartheta(i) = 0$) 'shadow banks', and banks with (at least some) insured deposits ($\vartheta(i) > 0$) 'commercial banks'. The sets of commercial- and shadow banks respectively are denoted by $\mathcal{I}_{CB}$ and $\mathcal{I}_{SB}$. The (relative) sizes of the two sectors are given by $D(\mathcal{I}_{CB})$ and $D(\mathcal{I}_{SB})$, with $D(\mathcal{I}_{CB}) + D(\mathcal{I}_{SB}) = 1$. Without loss of generality, I restrict attention to allocations in which the share of insured deposits at all commercial banks is set to the same level $\vartheta_{CB} \geqslant \vartheta^{sr}$. Since insuring more deposits entails no social cost, it is always optimal to insure the maximum possible amount of deposits, that is:

$$\underbrace{\overbrace{D(\mathcal{I}_{CB})}^{\substack{\text{size of}\\ \text{CB cector}}}\ \vartheta_{CB}}_{\substack{\text{total face value of}\\ \text{insured deposits}}} = \theta \tag{10}$$

The fact that commercial banks are not susceptible to systemic runs in the optimal structure of the financial system (lemma 4.1) implies that the share of insured deposits at commercial banks ϑ_{CB} must be above a certain threshold:

Lemma 4.2. *$\vartheta_{CB} \geqslant \vartheta^{sr}$ is equivalent to $\vartheta_{CB} \geqslant 1 - \lambda^S$*

The proof is given in appendix A.1. The sketch of the proof goes as follows: If commercial banks are susceptible to runs, then shadow banks must be susceptible to runs as well. It follows that commercial banks are not susceptible to runs if and only if they are not susceptible to runs in a hypothetical situation where all banks liquidate their portfolios, in which case the liquidation price equals $p = \lambda^S$. The rest then follows from condition 2.

Combining (10) with lemma 4.2, and inserting $D(\mathcal{I}_{CB}) = 1 - D(\mathcal{I}_{SB})$, yields the following condition on the relative size of the shadow banking sector in the optimal structure of the financial system:

$$D(\mathcal{I}_{SB}) \geqslant \max\left\{1 - \frac{\theta}{1-\lambda^S}, 0\right\} = D_{SB}^{min}(\theta) \tag{11}$$

$D_{SB}^{min}(\theta)$ equals the smallest size of the shadow banking sector at which the shadow banking sector is large enough to absorb enough of the uninsurable deposits from the commercial banking sector, such as to keep the commercial banking sector shielded from systemic runs. $D_{SB}^{min}(\theta)$ is decreasing in the share of insurable deposits θ. By condition 2 and expression 6, shadow banks are susceptible to systemic runs iff $p^{sr} < 1$. Given that commercial banks are not susceptible to systemic runs, this is the case iff the size of the shadow banking sector satisfies $D(\mathcal{I}_{SB}) > \lambda^S = D_{SB}^{max}$. Hence D_{SB}^{max} denotes the maximum size of the shadow banking sector at which the shadow banking sector is not susceptible to systemic runs. The discussion in the previous paragraphs leads us to the following proposition:

Proposition 4.1. *The optimal (relative) size of the shadow banking sector D_{SB}^{opt} depends on the deposit insurance cap θ as follows:*

(i) If $\theta \in [(1-\lambda^S), 1]$, then $D_{SB}^{opt} \in [0, D_{SB}^{max}]$

(ii) If $\theta \in [(1-\lambda^S)^2, (1-\lambda^S)]$, then $D_{SB}^{opt} \in [D_{SB}^{min}(\theta),\ D_{SB}^{max}]$

(iii) If $\theta \in [0, (1-\lambda^S)^2]$, then $D_{SB}^{opt} = D_{SB}^{min}(\theta)$

According to proposition 4.1, the deposit insurance cap θ can be divided into three regions. The three regions are also illustrated in Figure 4 further below, which depicts how the optimal size of the shadow banking sector (green line/area) depends on the cap θ, for an economy with secondary market capacity $\lambda^S = 0.25$. The dotted line in figure 4 is the $45°$-line. Note that if no deposits are insurable ($\theta = 0$), then the relative size of the shadow banking sector equals 1 by definition.

Consider first region (i) in which the cap is at a relatively high level. From condition 11 we get that $D_{SB}^{min}(\theta) = 0$, meaning that commercial banks are not susceptible to systemic runs even if all uninsurable deposits remain in the commercial banking sector. Systemic runs do not occur in the economy as long as the size of the shadow banking sector is within $[0, D_{SB}^{max}]$.[18]

Consider next the case where the cap is within the intermediate region (ii). For $\theta < (1-\lambda^S)$ we have that $D_{SB}^{min}(\theta) = 1 - \dfrac{\theta}{1-\lambda^S} > 0$, which means that the relative size of the shadow banking sector must be larger than zero in order to absorb enough uninsurable deposits from the commercial banking sector. On the other hand, we have that $D_{SB}^{min}(\theta) \leqslant D_{SB}^{max}$, with strict inequality for $\theta > (1-\lambda^S)^2$. This means that systemic runs can be avoided in both the commercial- and the shadow banking sector by setting the relative size of the shadow banking within $[D_{SB}^{min}(\theta), D_{SB}^{max}]$. If the cap is within region (ii), systemic runs can therefore be avoided by setting the shadow banking sector large enough to keep commercial banks shielded from systemic runs, but not too large, so that the shadow banking sector itself is not susceptible to systemic runs either.

[18] Within this region of the cap, we have that $1 - \theta < D_{SB}^{max}$, which means that the shadow banking sector is not susceptible to systemic runs even if it is larger than the share of uninsurable deposits. Hence the constraint (10) may be slack within this region of θ.

Lastly, consider region (iii) in which the cap is at a relatively low level. For $\theta < (1-\lambda^S)^2$, we have that $D_{SB}^{min}(\theta) > D_{SB}^{max}$. This means that the smallest size of the shadow banking sector at which the commercial banking sector is not susceptible to systemic runs is such that shadow banks are susceptible to systemic runs. At this level of the cap it is not feasible to avoid systemic runs altogether. The magnitude of systemic runs is minimized by setting the shadow banking to the smallest size necessary to absorb enough uninsurable deposits from the commercial banking sector.

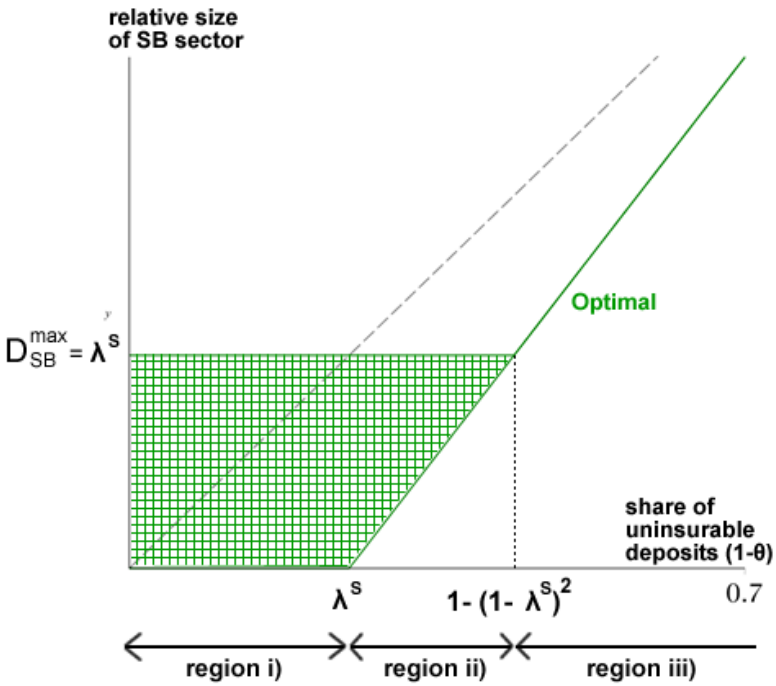

Figure 4: Optimal relative size of the shadow banking sector.

5 Competitive Equilibrium

In a competitive allocation, banks sell demand deposits to households in period 0. A demand deposit contract offered by a bank i stipulates the return $r(i)$ which the bank pays at the end of period 1 per unit of good invested into the bank in period 0. Denote $\tilde{\mu}(i)$ as the amount of good invested in intermediary i by a (representative) household.[19] Then $\tilde{\delta}(i) = \tilde{\mu}(i)\, r(i)$ denotes the face

[19]The indices for households are omitted throughout. Throughout the paper, I limit attention to symmetric equilibria, where symmetric means that all households make the same portfolio choice in period 0.

value of deposits held at bank i by the household. Households (depositors) can withdraw the full amount $\tilde{\delta}(i)$ already at the beginning of period 1. When buying deposits from banks, households choose for which deposits to obtain deposit insurance. Denote $\tilde{\vartheta}(i) \in [0,1]$ as the fraction of deposits held at bank i for which a household obtains deposit insurance. Household's portfolio choice in period 0 can then be expressed as choosing two integrable functions $(\tilde{\delta}(i), \tilde{\vartheta}(i))$ subject to the budget constraint and the cap on insured deposits.

An insured deposit held at bank i pays a riskless return of $r(i)$. An uninsured deposit held at bank i pays a riskless return $r(i)$ if the bank is not susceptible to runs. If the bank is susceptible to runs, then the effective return to an uninsured deposit is risky and depends on the realization of the sunspot variable $\xi \in \{0,1\}$. I again assume that, by some law of large numbers, households diversify away any idiosyncratic risk regarding the order of the line at individual banks.[20] Motivated by this (and in order to circumvent issues of measurability), households do not take into account idiosyncratic risk regarding the place in the line when making their portfolio decision; the return to an uninsured deposit in case of a systemic run ($\xi = 1$) is taken to be equal to the expected return, that is, $r(i)$ times the probability that the deposit can be withdrawn in the run. Households' utility maximization problem is then to choose any portfolio $(\tilde{\delta}(i), \tilde{\vartheta}(i))$ solving:[21]

$$\max_{(\tilde{\delta}(i),\tilde{\vartheta}(i))} \overbrace{E\left[u(c_1(\xi)\right]}^{\text{expected utility}} \quad \text{subject to} \quad \overbrace{\int_{\mathcal{I}} \frac{\tilde{\delta}(i)}{r(i)}\, di = 1}^{\text{budget constraint}} \quad \text{and} \quad \overbrace{\int_{\mathcal{I}} \tilde{\vartheta}(i)\, \tilde{\delta}(i)\, di \leqslant \theta}^{\text{cap on deposit insurance}} \tag{12}$$

A (symmetric) equilibrium is a tuple $(\tilde{\delta}(i), \tilde{\vartheta}(i), \delta(i), \vartheta(i), r(i))$ so that: (i) households' portfolio choice $(\tilde{\delta}(i), \tilde{\vartheta}(i))$ is such that households maximize expected utility according to (12); (ii) deposit contracts $r(i)$ offered by banks are such that no bank has a profitable deviation and (iii) the aggregate structure of the financial system corresponds to individual choices, $(\tilde{\delta}(i), \tilde{\vartheta}(i)) = (\delta(i), \vartheta(i))$.

[20] Intuitively, the return received from a portfolio of uninsured deposits in a systemic run by an individual household h should be given by $\int_{\mathcal{I}} I(i)^h (1 - \tilde{\vartheta}(i))\, \tilde{\delta}(i)\, di$ where $I(i)^h$ is random and takes the value 1 if household h is early in the line at bank i and can withdraw her deposits (or if bank i is not susceptible to runs) and 0 if household h is late in the line at bank i. Since the order of the line is random and independent at each bank, the sample path $I(i)^h$ is generally not measurable. See, for instance, Uhlig (1996) and Al-Najjar (2004) for possible remedies.

[21] Since households attach no value to consumption in period 0, and utility is strictly increasing in period 1 consumption, it is without loss of generality to set the budget constraint to equality.

First we can note that, since deposits at different banks are perfect substitutes, competition among banks will drive banks' profits to zero in equilibrium. This implies that the return paid by banks in equilibrium equals the return to the investment technology, that is, $r(i) = 1$ for all banks in equilibrium. This follows from the fact that banks face an infinitely elastic demand for insured deposits and, by the usual argument of Bertrand, the only equilibrium is one in which all banks offer $r(i) = 1$.[22] The face value of deposits issued by any bank thus corresponds to the fundamental value of assets held by the bank. Denoting $T(\xi)$ as the lump-sum payments to deposit insurance in state of nature ξ, we can then write down the expressions for consumption levels in the two states $\xi \in \{0, 1\}$:

$$\begin{aligned}
\underbrace{c_1(0)}_{\substack{\text{consumption}\\ \text{if no run}}} &= \underbrace{\int_{\mathcal{I}} \tilde{\delta}(i)\, di}_{\substack{\text{total face value}\\ \text{of deposits}}} - \underbrace{T(0)}_{\substack{\text{tax to deposit}\\ \text{insurance (=0)}}} \\
\underbrace{c_1(1)}_{\substack{\text{consumption}\\ \text{in systemic run}}} &= \int_{\mathcal{I}} \Big[\underbrace{\tilde{\vartheta}(i)}_{\substack{\text{fraction of}\\ \text{insured}\\ \text{deposits}\\ \text{at bank } i}} + \underbrace{(1-\tilde{\vartheta}(i))}_{\substack{\text{fraction of}\\ \text{uninsured}\\ \text{deposits}\\ \text{at bank } i}} \underbrace{\varphi(\vartheta(i), p^{sr})}_{\substack{\text{probability that}\\ \text{uninsured deposit at}\\ \text{bank } i \text{ can be withdrawn}\\ \text{in run (see 7)}}} \Big] \underbrace{\tilde{\delta}(i)}_{\substack{\text{total}\\ \text{deposits}\\ \text{at bank } i}} di - \underbrace{T(1)}_{\substack{\text{tax to}\\ \text{deposit}\\ \text{insurance}}}
\end{aligned} \tag{13}$$

The key difference between the competitive equilibrium and the welfare maximization problem (9) is that households take as given the aggregate structure of the financial system $(\delta(i), \vartheta(i))$ and aggregate financial stability (represented by the liquidation price p^{sr}) when making their portfolio decision in period 0.[23]

If systemic runs can occur in the economy $(p^{sr} < 1)$, then uninsured deposits at banks with a higher share of insured deposits dominate uninsured deposits at banks with a lower share of insured deposits. The reason is that $\varphi(\cdot)$ is increasing in $\vartheta(i)$, that is, the probability that an uninsured deposit can be withdrawn in a run is increasing in the share of insured deposits held at the bank.

[22] When deciding at which banks to hold their uninsured deposits, depositors prefer banks offering a higher return $r(i)$ to those offering a lower return as well, if the banks are otherwise identical (that is, if the fraction of insured and uninsured deposits held at the two banks is the same). This implies that, in a situation where all banks offer $r(i) = 1$, offering a lower return does not constitute a profitable deviation since this would not allow to attract any deposits (insured or uninsured). It also implies that $r(i) = 1$ for all banks in equilibrium if there is no deposit insurance ($\theta = 0$), by the same argument as above.

[23] The liquidation price p^{sr} is itself fully determined by the aggregate structure of the financial system $(\delta(i), \vartheta(i))$. See expression (6) and lemma 3.1.

(This results from the fact that insured deposits are not withdrawn in a run). If the share of insured deposits is above ϑ^{sr}, then the bank will not be affected by the systemic run at all. (Lemma 3.1). If systemic runs cannot occur in the economy $(p^{sr} = 1)$, then all deposits pay the same (riskless) return, independent of the share of insured and uninsured deposits at a bank. It follows that, when choosing how to invest the uninsurable part of their endowment, households weakly prefer banks with a higher share of insured deposits.[24] In particular, households never have an incentive to invest into 'shadow banks' with no insured depositors. This leads us to the following proposition:

Proposition 5.1. *In equilibrium it holds that:*

(i) Either all banks are susceptible to systemic runs or none are.

(ii) If systemic runs occur, then the share of insured deposits equals $\vartheta(i) = \theta$ for all banks.

Since households never have a strict incentive to invest into shadow banks, it is a somewhat trivial result that the size of the shadow banking sector in the competitive allocation is (weakly) smaller than the optimal size as given in proposition 4.1. From section 4 we know that it is not feasible to avoid systemic runs if the cap on insured deposits (θ) satisfies $\theta < (1 - \lambda^S)^2$. It thus follows from proposition 5.1 that the shadow banking sector is strictly smaller than the optimal size if $\theta < (1-\lambda^S)^2$. At this level of the cap, the optimal structure of the financial system features a shadow banking sector prone to systemic runs. This does not constitute a competitive equilibrium of the economy because households would have an incentive to move uninsured (and uninsurable) deposits from the unstable shadow banking sector into the stable commercial banking sector, thereby causing the commercial banking to become susceptible to systemic runs as well.[25]

[24] Note also that it is always (weakly) optimal for households to hold the maximum possible amount in insured deposits, given that deposit insurance entails no fee. If systemic runs occur, then uninsured deposits are risky and it is strictly optimal to hold the maximum amount θ in insured deposits.

[25] If the cap is within the region $\theta \in [(1 - \lambda^S)^2, (1 - \lambda^S)]$, then systemic runs do not occur in the economy if the shadow banking sector is at the "right size" (see section 4). If systemic runs do not occur, returns paid by banks do not depend on the share of insured deposits $\vartheta(i)$. This implies that a situation in which the shadow banking sector is "accidentally" at the right size so that systemic runs do not occur, constitutes an equilibrium of the economy if $\theta \in [(1 - \lambda^S)^2, (1 - \Lambda)]$. Since households never have a strict incentive to invest into shadow banks, it is questionable how plausible this equilibrium is. In any case, there is also an equilibrium in which shadow banks do not exist and systemic runs affect the entire financial system if $\theta \in [(1 - \lambda^S)^2, (1 - \lambda^S)]$.

6 Fees on Commercial Bank Deposits

The setting is now modified in the following way. Before posting deposit contracts, banks need to decide whether to get access to deposit insurance. If a bank decides not to get access to deposit insurance, it will be labelled a 'shadow bank' and households cannot obtain deposit insurance for the deposits issued by the bank. Banks that decide to get access to deposit insurance will be labelled 'commercial banks'. Households can, but do not have to, obtain deposit insurance for deposits issued by commercial banks. The cap on deposit insurance applies as before. Deposit insurance charges a fee on all deposits issued by commercial banks, insured or uninsured. The setting studied in this section is motivated by real-world institutional features; many deposit insurance schemes require commercial banks to pay a fee on deposits.[26] The focus of this section is to derive the structure of the financial system that results in a competitive equilibrium under this institutional framework, and compare it to the optimal structure of the financial system as derived in section 4. The main difference compared to section 5 is that households now have an incentive to hold deposits at shadow banks instead of commercial banks in order to avoid the fee charged on commercial bank deposits.

The fee on commercial bank deposits equals a fraction τ of the face value of deposits and is charged directly on households after they have withdrawn the deposit from the bank. The results regarding the optimal size of the shadow banking sector derived in section 4 are not affected by the fee.[27] Deposit insurance payments can now be seen as being partly financed by fee revenue and partly by lump-sum taxes if the fee revenue is not sufficient. Any fee revenue not used for deposit insurance payments is rebated to households in a lump-sum fashion at the end of period 1.

The definition of the equilibrium in the economy with a fee on commercial bank deposits is analogous to section 5 except for the explicit distinction between commercial banks and shadow banks.

[26] In the U.S., all bank liabilities are included in the assessment base used to determine the fees banks need to pay to the FDIC. Before the Dodd-Frank Act, the assesment base was total domestic deposits.

[27] In particular, the fee is set up in such a way that it does not affect the face value of outstanding deposits in period 1. If the fee were charged in period 0, or if it were charged on banks rather than directly on households, the aggregate fee revenue would affect the aggregate face value of outstanding deposits in period 1, even if by very little.

Competition among banks again implies that all banks offer $r(i) = 1$ in equilibrium. Insured deposits at commercial banks pay a riskless return of $(1-\tau)r(i) = (1-\tau)$. Uninsured deposits at commercial banks pay a return $(1-\tau)$ if they can be withdrawn and zero otherwise. Shadow bank deposits pay a return $r(i) = 1$ if they can be withdrawn from the bank and zero otherwise. Analogous to before, the subsets of banks operating as commercial banks and shadow banks are denoted by $\mathcal{I}_{CB}$ and $\mathcal{I}_{SB}$ respectively and the (relative) sizes of the two sectors are given by $D(\mathcal{I}_{CB})$ and $D(\mathcal{I}_{SB})$. I will sometimes say that a given sector is 'unstable' if it is susceptible to systemic runs and 'stable' if it is not.

In a systemic run, a commercial bank $i \in \mathcal{I}_{CB}$ can pay out a fraction $\varphi(\vartheta(i), p^{sr})$ of its uninsured deposits, where $\varphi(\cdot)$ is increasing in the share of insured deposits $\vartheta(i)$ and the liquidation price p^{sr} (see (7)). A shadow bank can pay out a fraction $\varphi(0, p^{sr}) = p^{sr}$ of deposits in case of a systemic run. When choosing at which type of bank to hold uninsured deposits, households trade off the fee τ charged on commercial bank deposits against higher losses caused by runs at shadow banks. We then immediately get the following result:

Lemma 6.1. *In the economy with a fee on commercial bank deposits, there is no equilibrium with stable shadow banking.*

The proof goes as follows. Suppose there is an equilibrium with stable shadow banks, that is, with shadow banks that are not susceptible to systemic runs. Then both shadow- and commercial bank deposits are riskless. Since shadow bank deposits do not entail the fee τ, they dominate commercial bank deposits (both insured and uninsured). This means that households invest all endowment into shadow banks. But if all endowment is invested into shadow banks, shadow banks are susceptible to systemic runs, which follows from limited secondary market capacity ($\lambda^S < 1$). Hence we have a contradiction.

If follows from lemma 6.1 that we only need to consider two types of equilibria:

(i) Equilibria in which a stable commercial banking sector coexists with a shadow banking sector susceptible to systemic runs. (Labelled **type A** equilibria).

(ii) Equilibria in which systemic runs affect all banks. (Labelled **type B** equilibria).

Before proceeding, I add the following assumption about parameters:

Assumption 6.1. $0 < \tau < (1 - \lambda^S)\,\pi^r$

Assumption 6.1 puts an upper bound on the fee on commercial bank deposits. The upper bound on τ is such that, if systemic runs affect the entire financial system (implying $p^{sr} = p(1) = \lambda^S$), then the riskless return to insured commercial bank deposits is higher than the expected return to shadow bank deposits.[28] Assumption 6.1 hence implies that households prefer *insured* commercial bank deposits to shadow bank deposits if financial stability is at the lowest possible level.

Type A equilibria In a type A equilibrium, only shadow banks are susceptible to systemic runs. Since commercial banks are not susceptible to runs, both insured and uninsured commercial bank deposits pay a riskless return of $r_{CB} = 1 - \tau$. The amount of assets sold in a systemic run increases in the size of the shadow banking sector. The liquidation price of assets in a systemic run in a type A equilibrium is denoted p_A^{sr} and, due to cash-in-the-market pricing, is decreasing in the size of the shadow banking sector: $p_A^{sr} = p(D(\mathcal{I}_{SB})) = \dfrac{\lambda^S}{D(\mathcal{I}_{SB})} < 1$ (see 1). As described earlier, p^{sr} also equals the fraction of deposits that shadow banks can serve in a systemic run. I again assume that, by a law of large numbers, idiosyncratic risk regarding the order in the line at individual (shadow-) banks in case of a systemic run is diversified away. Denote $\hat{r}_{SB,A}(D(\mathcal{I}_{SB}), \xi)$ as the effective return

[28] In a systemic run, shadow banks can pay out a fraction $\varphi(0, p^{sr}) = p^{sr}$ of deposits. The probability of a systemic run equals π^r. Hence the ex ante expected return to shadow bank deposits, given that $p^{sr} = \lambda^S$, equals $(1-\pi^r)+\pi^r\lambda^S$. Assumption 6.1 can be rewritten as $1 - \tau > (1 - \pi^r) + \pi^r\lambda^S$.

on a portfolio of shadow bank deposits in a type A equilibrium in state of nature ξ. We have:

$$\overbrace{\hat{r}_{SB,A}(D(\mathcal{I}_{SB}),\xi)}^{\substack{\text{effective return on}\\ \text{shadow bank deposits}}} = \begin{cases} 1 & \text{if } \xi = 0 \text{ (no run)} \\ \dfrac{\lambda^S}{D(\mathcal{I}_{SB})} = p_A^{sr} & \text{if } \xi = 1 \text{ (run)} \end{cases} \tag{14}$$

Losses caused by systemic runs on the shadow banking sector are increasing in the size of the shadow banking sector, which implies that the relative attractiveness of shadow bank deposits decreases as the shadow banking sector grows.[29] Denote $\tilde{\alpha}_A \in [0,1]$ as the share of a household's deposits held at shadow banks, with the remaining fraction $(1-\tilde{\alpha}_A)$ held at commercial banks. Households optimally choose $\tilde{\alpha}_A$, taking as given the aggregate structure of the financial system. Expected utility is continuous, as well as strictly concave in $\tilde{\alpha}_A$ (see appendix B). By the theorem of the maximum, households' optimal investment into the shadow banking sector, denoted $\tilde{\alpha}_A^{opt}$, can be expressed as a continuous, decreasing function of the size of the shadow banking sector $D(\mathcal{I}_{SB})$. We get the following result:

Lemma 6.2. *There exists a threshold size of the shadow banking sector $\underline{D}_{SB} \geqslant \dfrac{\lambda^S}{1-\tau}$ so that households' optimal choice is $\tilde{\alpha}_A^{opt} = 1$ (all deposits held at shadow banks) if and only if $D(\mathcal{I}_{SB}) \leqslant \underline{D}_{SB}$. Furthermore, there exists a threshold $\overline{D}_{SB} \leqslant 1$ so that households' optimal choice is $\tilde{\alpha}_A^{opt} = 0$ (all deposits held at commercial banks) if and only if $D(\mathcal{I}_{SB}) \geqslant \overline{D}_{SB}$.*

The formal proof of lemma 6.2 is given in appendix A.2. The intuition goes as follows: The fact that shadow banks are susceptible to systemic runs implies that the size of the shadow banking sector satisfies $D(\mathcal{I}_{SB}) > \lambda^S$ in any type A equilibrium. However, as $D(\mathcal{I}_{SB})$ approaches λ^S from above, losses caused by runs on shadow banks go to zero (see 14). By the same reasoning as in lemma 6.1 this implies that it is optimal to invest only in shadow banks ($\tilde{\alpha}_A^{opt} = 1$) if the relative size of the shadow banking sector is higher, but very close to, λ^S. On the other hand, the upper bound on the fee τ in assumption 6.1 implies that it is optimal to invest only in commercial banks ($\tilde{\alpha}_A^{opt} = 0$) as

[29] The only other effect of an increase in the size of the shadow banking sector from the point of view of an individual household is that the fee revenue rebated by the deposit insurance agency decreases as a result of the smaller size of the commercial banking sector, which affects consumption in all states identically. Note also that, since systemic runs only encompass the shadow banking sector, deposit insurance never needs to make payments in a type A equilibrium.

the relative size of the shadow banking sector approaches one. To summarize, households' optimal choice, given that the economy is in a type A equilibrium, satisfies:

$$\tilde{\alpha}_A^{opt}(D(\mathcal{I}_{SB})) = \begin{cases} 1 & \text{if } D(\mathcal{I}_{SB}) \leqslant \underline{D}_{SB} \\ \substack{\text{continuous} \\ \text{and decreasing} \\ \text{in } D(\mathcal{I}_{SB})} & \text{if } D(\mathcal{I}_{SB}) \in [\underline{D}_{SB}, \overline{D}_{SB}] \\ 0 & \text{if } D(\mathcal{I}_{SB}) \geqslant \overline{D}_{SB} \end{cases} \tag{15}$$

By market clearing, we have that $D(\mathcal{I}_{SB}) = \tilde{\alpha}_A^{opt}$ in equilibrium. Hence we can express households' optimal choice as a continuous and decreasing function $\tilde{\alpha}_A^{opt}(\overbrace{D(\mathcal{I}_{SB})}^{\tilde{\alpha}_A})$ mapping $[0, 1]$ onto itself. It follows that there is a unique fixed point $\tilde{\alpha}_A^* = \tilde{\alpha}_A^{opt}(\tilde{\alpha}_A^*)$, and $D_{SB,A}^* = \tilde{\alpha}_A^*$ is the size of the shadow banking sector in the only candidate for a type A equilibrium. The size of the shadow banking sector in a type A equilibrium is such that households are indifferent at the margin between investing into the shadow banking sector, which is prone to systemic runs, and paying the fee τ on commercial bank deposits. Since $\tilde{\alpha}_A^{opt}(0) = 1$ and $\tilde{\alpha}_A^{opt}(1) = 0$, corner solutions are ruled out and we have $D_{SB,A}^* \in (\underline{D}_{SB}, \overline{D}_{SB})$. It remains to check whether, at $D(\mathcal{I}_{SB}) = D_{SB,A}^*$, the economy is indeed in a type A equilibrium, that is, in a situation in which shadow banks but not commercial banks are susceptible to runs, as has been presumed in (15). From lemma 4.2, it follows that commercial banks are *not* susceptible to systemic runs if and only if the share of insured deposits at commercial banks satisfies $\vartheta_{CB} \geqslant 1 - \lambda^S$. We have that:

$$\underbrace{\vartheta_{CB}}_{\substack{\text{share of insured} \\ \text{deposits at} \\ \text{commercial banks}}} \leqslant \frac{\overbrace{\theta}^{\substack{\text{total} \\ \text{insurable} \\ \text{deposits}}}}{\underbrace{D(\mathcal{I}_{CB})}_{\substack{\text{total deposits at} \\ \text{commercial banks}}}} \tag{16}$$

Inserting $D(\mathcal{I}_{CB}) = 1 - D(\mathcal{I}_{SB})$ into condition (16) we get that, given the relative size of the shadow banking sector equals $D(\mathcal{I}_{SB}) = D_{SB,A}^*$, a stable commercial banking sector is feasible if and only if the share of insurable deposits satisfies:

$$\theta \geqslant (1 - \lambda^S)(1 - D_{SB,A}^*) \equiv \underline{\theta}_A \tag{17}$$

It follows that $D(\mathcal{I}_{SB}) = D^*_{SB,A}$ constitutes a type A equilibrium if and only if the cap satisfies $\theta \geqslant \underline{\theta}_A$. Intuitively, if $\theta < \underline{\theta}_A$, then the economy does not exhibit a type A equilibrium because the share of insurable deposits is too low to allow for a stable commercial banking sector. The preceding discussion leads us to the following proposition:

Proposition 6.1. *The economy exhibits a type A equilibrium if and only if the share of insurable deposits satisfies $\theta \geqslant \underline{\theta}_A$.*

The comparative statics regarding the relative size of the shadow banking sector in the type A equilibrium ($D^*_{SB,A}$) are rather straightforward: All else equal, $D^*_{SB,A}$ increases in the fee on commercial bank deposits (τ) as well as secondary market capacity (λ^S), and decreases in the probability of systemic runs (π^r) as well as the degree of households' risk aversion. I refer to appendix B for a formal derivation of the comparative statics results. Note that changes in the deposit insurance cap θ have no effect on the size of the shadow banking sector in a type A equilibrium as long as θ stays above $\underline{\theta}_A$. If θ decreases below $\underline{\theta}_A$, the economy moves to a type B equilibrium, with an ambiguous effect on the size of the shadow banking sector (see below).

Type B equilibria A type B equilibrium is an equilibrium in which all banks are susceptible to systemic runs. This means that the total fundamental value of assets sold in a systemic run equals $\lambda^D = 1$ and the liquidation price of assets in a systemic run equals $p^{sr}_B = p(1) = \lambda^S$ (see 1). Different to a type A equilibrium, the magnitude of systemic runs does not depend on the size of the shadow banking sector.

Insured commercial bank deposits pay a riskless return of $1-\tau$. The upper bound on τ (assumption 6.1) means that, in a situation in which systemic runs affect the entire financial system, insured commercial bank deposits are preferred to uninsured deposits at any bank. Hence, different to a type A equilibrium, in a type B equilibrium households will always hold the maximum possible amount (θ) in insured commercial bank deposits. This implies that condition (16) is binding in a type B equilibrium. The relevant choice of households is therefore how to allocate the uninsurable part of their deposits between commercial banks and shadow banks. When deciding how to allocate

the uninsurable part of their deposits between commercial banks and shadow banks, households again trade off the fee charged on uninsured commercial bank deposits against lower losses caused by runs due to the presence of insured depositors.

Given that the share of insured deposits at commercial banks equals ϑ_{CB}, the fraction of uninsured deposits that can be withdrawn from commercial banks in a systemic run equals $\varphi(\vartheta_{CB}, \lambda^S) = \dfrac{\lambda^S}{1-\vartheta_{CB}}$ (see 7). Denote $\hat{r}_{CB,B}(\vartheta_{CB}, \xi)$ as the effective return to a portfolio of uninsured commercial bank deposits in a type B equilibrium in state of nature ξ. We have:

$$\overbrace{\hat{r}_{CB,B}(\vartheta_{CB}, \xi)}^{\substack{\text{effective return on}\\ \text{uninsured commercial-}\\ \text{bank deposits}}} = \begin{cases} 1-\tau & \text{if } \xi = 0 \text{ (no run)} \\ \overbrace{\varphi(\vartheta_{CB}, \lambda^S)}^{\substack{\text{increasing}\\ \text{in } \vartheta_{CB}}} (1-\tau) & \text{if } \xi = 1 \text{ (run)} \end{cases} \tag{18}$$

The effective return to a portfolio of shadow bank deposits in a type B equilibrium is given by:

$$\overbrace{\hat{r}_{SB,B}(\xi)}^{\substack{\text{effective return}\\ \text{on shadow bank}\\ \text{deposits}}} = \begin{cases} 1 & \text{if } \xi = 0 \text{ (no run)} \\ \overbrace{\varphi(0, \lambda^S)}^{=\lambda^S} & \text{if } \xi = 1 \text{ (run)} \end{cases} \tag{19}$$

Note that uninsured commercial bank deposits are the more attractive relative to shadow bank deposits the higher the share of insured deposits in the commercial banking sector, ϑ_{CB}. Denote the share of a household's uninsurable deposits invested into shadow banks by $\tilde{\alpha}_B \in [0, 1]$. Households' expected utility is continuous, as well as strictly concave in $\tilde{\alpha}_B$ (see appendix C). By the theorem of the maximum, households' optimal choice, denoted $\tilde{\alpha}_B^{opt}$, can be expressed as a continuous and decreasing function of ϑ_{CB}. We get the following result:

Lemma 6.3. *There exists a threshold share of insured deposits in the commercial banking sector $\underline{\vartheta} \geqslant \tau$ so that households choose $\tilde{\alpha}_B^{opt} = 1$ (all uninsured deposits held at shadow banks) if and only if $\vartheta_{CB} \leqslant \underline{\vartheta}$. Furthermore, there exists a threshold $\overline{\vartheta} \leqslant 1 - \lambda^S$ so that households choose $\tilde{\alpha}_B^{opt} = 0$ (all uninsured deposits held at commercial banks) if and only if $\vartheta_{CB} \geqslant \overline{\vartheta}$.*

The formal proof of lemma 6.3 is given in appendix A.3. The intuition goes as follows. The fact that commercial banks are susceptible to systemic runs implies that $\vartheta_{CB} < 1 - \lambda^S$ in any type B equilibrium (lemma 4.2). However, as ϑ_{CB} approaches $1 - \lambda^S$ from below, actual losses caused by runs for uninsured depositors at commercial banks go to zero (see 18). Hence for ϑ_{CB} below, but very close to, $1-\lambda^S$ it is optimal to hold all uninsured deposits at commercial banks ($\tilde{\alpha}_B^{opt} = 1$). On the other hand, in the limit as ϑ_{CB} approaches zero from above, losses caused by runs at commercial banks are the same as on shadow banks, while commercial bank deposits entail the fee τ. Hence for ϑ_{CB} close enough to zero, it is optimal to hold all uninsured deposits at shadow banks ($\tilde{\alpha}_B^{opt} = 0$). To summarize, households' optimal choice, given that the economy is in a type B equilibrium, satisfies:

$$\tilde{\alpha}_B^{opt}(\vartheta_{CB}) = \begin{cases} 1 & \text{if } \vartheta_{CB} \leqslant \underline{\vartheta} \\ \substack{\text{continuous} \\ \text{and decreasing} \\ \text{in } \vartheta_{CB}} & \text{if } \vartheta_{CB} \in [\underline{\vartheta}, \overline{\vartheta}] \\ 0 & \text{if } \vartheta_{CB} \geqslant \overline{\vartheta} \end{cases} \tag{20}$$

By market clearing, we have that $D(\mathcal{I}_{SB}) = (1-\theta)\,\tilde{\alpha}_B$ in equilibrium. By setting expression (16) to equality and inserting $D(\mathcal{I}_{SB}) = 1 - D(\mathcal{I}_{CB})$, we can express the share of insured deposits at commercial banks as an increasing function of households' investment into shadow banks: $\vartheta_{CB}(\tilde{\alpha}_B) = \dfrac{\theta}{1-(1-\theta)\tilde{\alpha}_B}$. Since shadow banks absorb uninsurable deposits from the commercial banking sector, the share of insured deposits at commercial banks is increasing in the size of the shadow banking sector. Hence a larger shadow banking sector implies lower losses caused by runs on commercial banks and therefore increases the relative attractiveness of uninsured commercial bank deposits compared to shadow bank deposits. Similar to the type A equilibrium, we can therefore conclude that optimal investment into shadow banks $\tilde{\alpha}_B^{opt}$ must be decreasing in the size of the shadow banking sector.[30] It follows that we can express households' optimal choice as a continuous and decreasing function $\tilde{\alpha}_B^{opt}(\tilde{\alpha}_B)$ mapping $[0,1]$ into itself. This means that there is a unique fixed point $\tilde{\alpha}_B^* = \tilde{\alpha}_B^{opt}(\tilde{\alpha}_B^*)$, which is the only candidate for a type B equilibrium. The size of the shadow

[30] As in the type A equilibrium, the only other effect of a larger shadow banking sector from the point of view of an individual household is that the fee revenue rebated from the deposit insurance agency decreases as a result of the smaller commercial banking sector. This affects consumption in all states identically. In case of a systemic run, the tax raised by deposit insurance equals the aggregate face value of insured deposits (θ), independent of the size of the shadow banking sector.

banking sector in the unique candidate for a type B equilibrium is given by $D^*_{SB,B} = (1-\theta)\tilde{\alpha}^*_B$. It then remains to check whether, at $D(\mathcal{I}_{SB}) = D^*_{SB,B}$, the economy is indeed in a type B equilibrium, that is, in a situation in which all banks are susceptible to systemic runs as has been presumed in (20). This is the case if and only if the share of insured deposits at commercial banks satisfies $\vartheta_{CB}(\tilde{\alpha}^*_B) < 1 - \lambda^S$ (lemma 4.2). We get the following result:

Proposition 6.2. *The economy exhibits a type B equilibrium if and only if the share of insurable deposits satisfies $\theta < 1 - \lambda^S$.*

The proof is given in appendix A.4. Intuitively, if the share of insurable deposits is relatively high ($\theta \geqslant 1-\lambda^S$), then there is no equilibrium in which systemic runs affect the entire financial system.

Next, I will show that if the cap θ is not too far below $1 - \lambda^S$, then only commercial banks (and no shadow banks) exist in a type B equilibrium. To see this, note first that, if all uninsured deposits are held at commercial banks ($\tilde{\alpha}_B = 0$) then the share of insured deposits at commercial banks equals $\vartheta_{CB}(0) = \theta$. Now suppose we have $\theta \in [\overline{\vartheta}, 1 - \lambda^S)$. Then it holds that $\overline{\vartheta} \leqslant \vartheta_{CB}(0) < 1 - \lambda^S$. This means that, in a situation in which all uninsurable deposits are held at commercial banks, systemic runs affect the entire financial system. At the same time, since $\vartheta_{CB}(0) \geqslant \overline{\vartheta}$, it is optimal for households to hold all uninsurable deposits at commercial banks in this situation. It follows that the economy exhibits a type B equilibrium with only commercial banks and no shadow banks if $\theta \in [\overline{\vartheta}, 1 - \lambda^S)$, where $\overline{\vartheta}$ may itself depend on θ. Intuitively, in a situation of low aggregate financial stability in which the entire financial system is prone to systemic runs, it can be privately optimal for households to hold all uninsured deposits at commercial banks rather than investing into even less stable shadow banks. This is only true if the cap on deposit insurance, and hence the share of insured deposits at commercial banks, is not all too low however. If $\vartheta_{CB}(0) = \theta < \overline{\vartheta}$, then the stability provided by the (low) share of insured deposits at commercial banks does not compensate for the fee on commercial bank deposits anymore and households will hold part of their uninsurable deposits in shadow banks. This leads us to the following proposition whose proof is given in appendix 6.3.

Proposition 6.3. *There exists a $\hat{\theta}_B$, with $0 < \hat{\theta}_B < 1 - \lambda^S$, so that the economy exhibits a type B equilibrium with only commercial banks if and only $\theta \in [\hat{\theta}_B, 1 - \lambda^S)$.*

Note next that, since $\underline{\theta}_A < 1 - \lambda^S$ (see 17), the economy exhibits both a type A and a type B equilibrium if the share of insurable deposits is within $\theta \in [\underline{\theta}_A, 1 - \lambda^S)$. Expected utility of households is higher in the type A equilibrium due to the smaller extent of systemic runs. Multiplicity of equilibria arises because households' optimal portfolio choice depends on aggregate financial stability (captured by the liquidation price p^{sr}) which in turn depends on households' portfolio choices in period 0.

To illustrate why the economy exhibits multiple equilibria for a certain range of the cap θ, suppose that the cap is within $\theta \in [\underline{\theta}_A, 1 - \lambda^S)$ and all endowment is invested into commercial banks. Then the share of insured deposits in the commercial banking sector equals $\vartheta_{CB} = \theta < 1 - \lambda^S$ and, by lemma 4.2, the commercial banking sector, and therefore the entire financial system, is prone to systemic runs. From proposition 6.3 we know that this situation may constitute a type B equilibrium of the economy.[31] Suppose now that, starting from an equilibrium with only commercial banks, a large part of the uninsured (and uninsurable) deposits is moved at once from the commercial banking sector into a newly created shadow banking sector. If a large enough part of uninsured deposits is moved from commercial banks into shadow banks, the share of insured deposits at commercial banks (ϑ_{CB}) will become higher than $1 - \lambda^S$, which implies that the commercial banking sector will not be prone to systemic runs anymore. The resulting increase in aggregate financial stability (captured by an increase in the liquidation price p^{sr}) lowers riskiness of both shadow bank deposits and uninsured commercial bank deposits. However the effect is more pronounced for shadow bank deposits, making shadow bank deposits relatively more attractive compared to uninsured commercial bank deposits. This reflects the fact that the change in p^{sr} has a one-to-one effect on the effective return to shadow bank deposits in a systemic run, while the effect on uninsured commercial bank deposits is mitigated by the fact that insured depositors at commercial banks do not participate in runs. As a result, given that aggregate financial stability (p^{sr}) has increased, it is now privately op-

[31] Whether this is true for the entire range $[\underline{\theta}_A, 1 - \lambda^S)$ depends on the position of $\hat{\theta}_B$ relative to $\underline{\theta}_A$, which depends on parameters.

timal for households to invest part of their endowment into shadow banks. The new situation with a shadow banking sector and a smaller extent of systemic runs therefore constitutes an equilibrium of the economy as well.

Figure (5) illustrates how the equilibrium of the economy with a fee on commercial bank deposits depends on the deposit insurance cap θ, according to propositions 6.1 and 6.2.

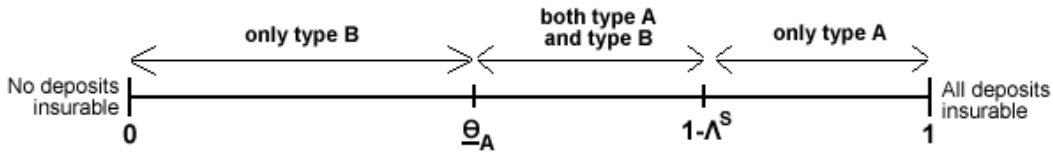

Figure 5: Equilibria of the economy depending on θ.

Appendices B and C describe how to solve for the type A and type B equilibria. Consider the following example:[32]

Example 6.1. $\overbrace{u(c) = ln(c)}^{\textit{utility of households}}$, $\overbrace{\lambda^S = 0.25}^{\textit{secondary market capacity}}$, $\overbrace{\pi^r = 0.2}^{\textit{probability of run}}$, $\overbrace{\tau = 0.05}^{\textit{fee on commercial bank deposits}}$

In example 6.1 we have that $\underline{\theta}_A = 0.51$, which means that the economy exhibits a type A equilibrium as long as the deposit insurance cap satisfies $\theta \geqslant \underline{\theta}_A = 0.51$. The relative size of the shadow banking sector in a type A equilibrium is given by $D^*_{SB,A} = 0.33$. For any $\theta < 1 - \lambda^S = 0.75$ the economy exhibits a type B equilibrium. This means that the economy exhibits multiple equilibria (type A and type B) if the cap is within $\theta \in [0.51, 0.75)$. We also get that $\hat{\theta}_B = 0.21$ so that shadow banks do not exist in the type B equilibrium if $\theta \in [0.21, 0.75)$.

Figure 6 shows the relative size of the shadow banking sector in the competitive equilibria compared to the the optimal size (as described in proposition 4.1) for the economy of example 6.1. As before, the X-axes show the share of uninsurable deposits in the economy. The dotted lines are the $45°$-lines. If the share of uninsurable deposits satisfies $1 - \theta \leqslant \lambda^S$, then the type A equilibrium is the only equilibrium of the economy and the shadow banking sector is larger than the optimal size. If the

[32] I do not attempt to calibrate the model. One might argue that a more realistic parametrization would involve a lower probability of a run and a higher degree of risk aversion. This would lead to quantitatively similar results.

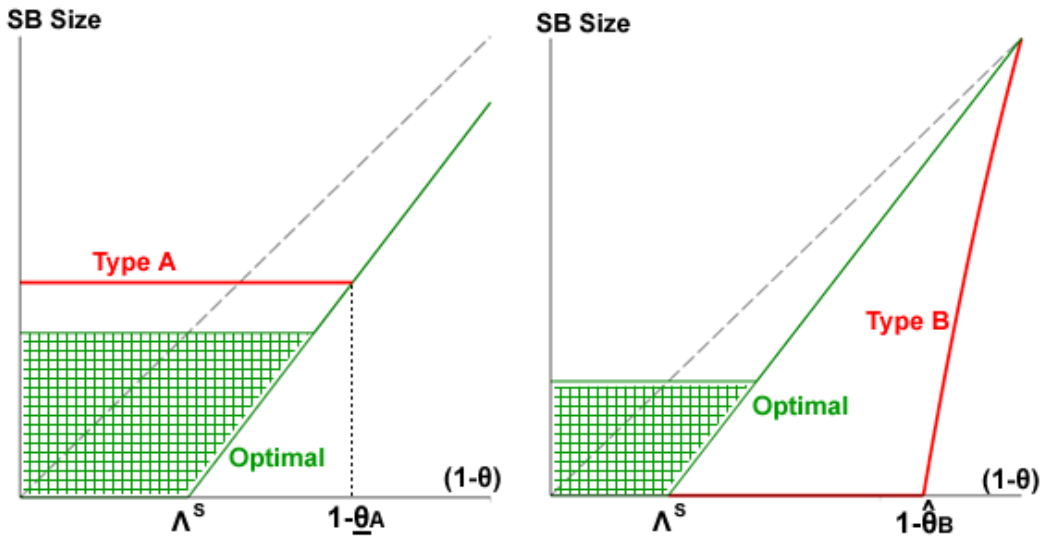

Figure 6: Size of shadow banking sector in competitive equilibrium vs. optimal size.

share of uninsurable deposits is at such a low level, then the optimal structure of the financial system features a relatively small shadow banking sector or no shadow banking sector at all. However, households have a private incentive to invest into shadow banks in order to avoid the fee imposed on commercial bank deposits. The shadow banking sector grows up to a size at which households are indifferent at the margin between investing in (stable) commercial banks and (unstable) shadow banks. We have the opposite situation if the share of uninsurable deposits satisfies $1-\theta > 1-\underline{\theta}_A$. If the share of uninsurable deposits is at such a high level, then the type B equilibrium is the only equilibrium of the economy and the shadow banking sector is smaller than the optimal size. The optimal structure of the financial system features an unstable shadow banking sector (of a size larger than in a hypothetical type A equilibrium) that coexists with a stable commercial banking sector. This allocation does not constitute an equilibrium due to households' private incentive to move uninsurable deposits from the unstable shadow banking sector into the stable commercial banking sector. If the share of uninsurable deposits is within the region $\theta \in [\lambda^S, 1-\underline{\theta}_A)$, then the shadow banking sector may be larger or smaller than the optimal size, depending on which equilibrium (type A or type B) is selected.[33]

[33] In the special case where $1-\theta = 1-\underline{\theta}_A$ and the economy is in a type A equilibrium, the size of the shadow banking sector corresponds to the optimal size.

In this section, I assumed that the deposit insurance agency charges a fee on all commercial bank deposits, insured or uninsured, which seems a reasonable description of current policies. Consider now an alternative situation where all banks have access to deposit insurance and the fee is only charged on insured deposits. A result equivalent to lemma 6.1 would still hold, implying that systemic runs occur in every competitive equilibrium. The reason is that households only have an incentive to pay the fee on insured deposits if systemic runs make an investment into uninsured deposits risky. In addition, given that systemic runs occur, households have an incentive to hold uninsured deposits at these intermediaries with the highest share of insured deposits (see the discussion in section 5). Hence the only equilibrium of this economy is an equilibrium where all banks have an identical share of insured deposits and systemic runs affect the entire financial system. The equilibrium of this economy essentially corresponds to item (ii) in proposition 5.1 except that the share of insured deposits at the representative commercial bank is endogenous and is such that households are indifferent at the margin between paying the fee for deposit insurance and bearing losses caused by runs on the representative bank.

7 Implementing the Optimal Size of the Shadow Banking Sector

It remains the question how a regulator can implement the optimal size of the shadow banking sector in a competitive equilibrium, taking as given the deposit insurance scheme in place. Consider first the setting of section 5, where obtaining deposit insurance is costless for households and the equilibrium size of the shadow banking sector is weakly smaller than the optimal size. Implementing the optimal size of the shadow banking sector requires that the share of uninsured deposits among all deposits held at banks that issue insured deposits ('commercial banks') be limited to λ^S. This can be achieved with a tax scheme that disincentivizes households to hold more than a fraction λ^S of their deposits purchased from commercial banks as uninsured deposits. For instance, the regulator could impose a marginal tax on all uninsured deposits that exceed a fraction λ^S of the deposits purchased at the same commercial bank. The tax must be such that households prefer shadow bank deposits to uninsured commercial bank deposits that are subject to the tax. Losses caused by sys-

temic runs in the shadow banking sector are increasing in the size of the shadow banking sector. The larger the size of the shadow banking sector, the higher the incentive for households to move deposits from the unstable shadow banking sector into the stable commercial banking sector. As a result, in order to keep the size of the shadow banking sector at its optimal level, the marginal tax on uninsured commercial bank deposits must be increasing in the share of uninsurable deposits $1 - \theta$.

Things get more complicated if the deposit insurance agency charges a fee on commercial bank deposits as in section 6. In this case, the shadow banking sector can be smaller or larger than the optimal size. A marginal tax on insured commercial bank deposits as described above ensures that the shadow banking sector is not too small relative to the optimal size, but it does not prevent the shadow banking sector from growing too large relative to the optimal size. The shadow banking sector can be larger than the optimal size due to households' private incentive to avoid the fee charged on commercial bank deposits. Preventing the shadow banking sector from growing too large requires that the regulator eliminate these incentive effects, for instance by charging a tax on all shadow bank deposits that is identical to the fee charged on commercial bank deposits by the deposit insurance agency. In the set-up of section 6, the optimal size of the shadow banking sector can thus be implemented with a two-pronged policy that consists of (i) a tax on shadow bank deposits that mimics any fee charged on commercial bank deposits and (ii) a marginal tax on uninsured commercial bank deposits that is charged whenever the amount of uninsured deposits held at a commercial bank exceeds a certain value.

8 Conclusion

This paper presents a theoretical argument why the distribution of insured and uninsured short-term claims across financial institutions matters for financial stability. If there is significant demand for short-term claims by investors with large endowments relative to the cap on deposit insurance,

the presence of a shadow banking sector that issues short-term claims which are not protected by deposit insurance may be beneficial from a financial stability point of view.

One of the main conclusions of this paper is that, in the context of limited deposit insurance, policies aimed at curtailing the shadow banking sector should be viewed with caution. This is especially true if such policies lead to a flow of uninsured deposits into the commercial banking sector.

Of course, these results are derived in a highly stylized environment. For instance, the commercial- and shadow banking sectors are modelled as essentially separated from each other, connected only via a common secondary market to which they both sell assets. In reality, there are tight interconnections between commercial- and shadow banks; many shadow banks are owned by parent companies that also own commercial banks and shadow banks are an important provider of short-term loans to commercial banks. Furthermore, this paper focuses on government-provided deposit insurance, abstracting from other government interventions such as the lender-of-last-resort or implicit bail-out guarantees to commercial banks. Finally, the underlying causes of the demand for short-term claims by investors with large endowments are not addressed. While such limitations must be kept in mind, the paper does shed light on some aspects of shadow banking in the context of limited deposit insurance that have not been analyzed so far.

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Appendix

Throughout the appendix, CB stands for 'commercial bank' and SB for 'shadow bank'.

A Proofs

A.1 Proof of lemma 4.2

First I show that $\vartheta_{CB} \geqslant \vartheta^{sr}$ implies $\vartheta_{CB} \geqslant 1 - \lambda^S$. Suppose $\vartheta_{CB} \geqslant \vartheta^{sr}$ and $\vartheta_{CB} < 1 - \lambda^S$. In a hypothetical situation in which all banks liquidate their portfolios, we have that $\lambda^D = 1$ so that the liquidation price equals $p(1) = \lambda^S$ (see 1). By (2), CBs are susceptible to runs in such a situation if $\vartheta_{CB} < 1 - \lambda^S$. Since SBs are always susceptible to runs if CBs are susceptible to runs (see 2) this implies that there is a systemic run equilibrium that affect all banks if $\vartheta_{CB} < 1 - \lambda^S$, which is a contradiction to $\vartheta_{CB} \geqslant \vartheta^{sr}$. Next, since the liquidation price cannot fall below $p(1) = \lambda^S$, it follows from (2) that CBs are never susceptible to systemic runs if $\vartheta_{CB} \geqslant 1 - \lambda^S$. Hence $\vartheta_{CB} \geqslant 1 - \lambda^S$ implies $\vartheta_{CB} \geqslant \vartheta^{sr}$ which completes the proof.

A.2 Proof of lemma 6.2

If $D(\mathcal{I}_{SB}) \leqslant \dfrac{\lambda^S}{1-\tau}$, then we have that $\hat{r}_{SB,A}(D(\mathcal{I}_{SB}), 1) \geqslant (1 - \tau)$. Hence CBs do not pay a higher effective return in case of a systemic run compared to SBs. Since SBs pay a higher return if no run takes place, SB deposits (first-order stochastically) dominate CB deposits, which implies that households' optimal choice is to invest only in SBs ($\tilde{\alpha}_A^{opt} = 1$). Suppose next that $D(\mathcal{I}_{SB}) = 1$. The expected return to SB deposits is then given by $E[\hat{r}_{SB,A}(D(\mathcal{I}_{SB}), \xi)] = \pi^r \lambda^S + (1 - \pi^r)$. By assumption 6.1 we have $\pi^r \lambda^S + (1 - \pi^r) < (1 - \tau)$, meaning that the expected return of (risky) SB deposits is lower than the riskless return to CB deposits. This implies that households' optimal choice is to invest only in CBs ($\tilde{\alpha}_A^{opt} = 0$). The rest follows from the fact that $\tilde{\alpha}_A^{opt}$ is continuous and decreasing in $D(\mathcal{I}_{SB})$.

A.3 Proof of lemma 6.3

If $\vartheta_{CB} \leqslant \tau$ then we have that $\hat{r}_{CB,B}(\vartheta_{CB}, 1) \leqslant \lambda^S$, which means that uninsured CB deposits do not pay a higher effective return in case of a systemic run than SB deposits. Since SB deposits pay a higher return if no run takes place, SB deposits first-order stochastically dominate uninsured CB deposits and households' optimal choice is to hold all uninsured deposits at SBs ($\tilde{\alpha}_B^{opt} = 1$). On the other hand we have that $\lim_{\vartheta_{CB} \nearrow (1-\lambda^S)} \varphi(\vartheta_{CB}) = 1$ (see expression 7), which means that actual losses for uninsured depositors caused by runs on CBs go to zero in the limit as ϑ_{CB} approaches $1 - \lambda^S$ from below. Hence we have that $\lim_{\vartheta_{CB} \nearrow (1-\lambda^S)} \hat{r}_{CB,B}(\vartheta_{CB}, 1) = 1 - \tau$. In the limit, uninsured CB deposits are riskless and pay a higher expected return than risky SB deposits, which implies that $\lim_{\vartheta_{CB} \nearrow (1-\lambda^S)} \tilde{\alpha}_B^{opt}(\vartheta_{CB}) = 0$. The rest follows from the fact that $\tilde{\alpha}_B^{opt}$ is continuous and decreasing in ϑ_{CB}.

A.4 Proof of proposition 6.2

As shown previously, there is a unique candidate $\tilde{\alpha}_B^*$ for a type B equilibrium for any given value of θ. First, I show that there cannot be a type B equilibrium if $\theta \geqslant 1 - \lambda^S$. Suppose $\theta \geqslant 1 - \lambda^S$ and the economy is in a type B equilibrium. Then we have $\vartheta_{CB}(\tilde{\alpha}_B) \geqslant \vartheta_{CB}(0) = \theta \geqslant 1 - \lambda^S$. By lemma 4.2, this implies that CBs are not susceptible to runs. Hence the economy is not in a type B equilibrium, which leads to a contradiction. Next, I show that there is a type B equilibrium if $\theta < 1 - \lambda^S$. For this it needs to be shown that, if $\theta < 1 - \lambda^S$, then $\vartheta_{CB}(\tilde{\alpha}_B^*) < 1 - \lambda^S$. Suppose $\theta < 1 - \lambda^S$ and $\vartheta_{CB}(\tilde{\alpha}_B^*) \geqslant 1 - \lambda^S$. Then, by (20), we have $\tilde{\alpha}_B^{opt} = \tilde{\alpha}_B^* = 0$ and the share of insured deposits at CBs is given by $\vartheta_{CB}(0) = \theta < 1 - \lambda^S$, which leads to a contradiction.

A.5 Proof of proposition 6.3

Since expected utility is strictly concave in $\tilde{\alpha}_B$ (see appendix C), we have that $\tilde{\alpha}_B = 0$ is the optimal choice for households if and only if

$$\frac{d\,E[u(c_1^B(\tilde{\alpha}_B, \xi))]}{d\,\tilde{\alpha}_B} \leqslant 0 \text{ at } \tilde{\alpha}_B = 0 \tag{21}$$

where $c_1^B(\tilde{\alpha}_B, \xi)$ denotes consumption at t=1 in a type B equilibrium in state $\xi \in \{0, 1\}$ and is given by (29). Market clearing implies that $\vartheta_{CB} = \dfrac{\theta}{1-(1-\theta)\tilde{\alpha}_B}$ (see section 6 in the main text). Inserting $\vartheta_{CB} = \theta$ (market clearing for $\tilde{\alpha}_B = 0$) into condition (21), and cancelling out all transfers to- and from deposit insurance, yields:

$$\frac{\theta - \tau}{1-\theta} \geqslant \frac{1-\pi^r}{\pi^r}\frac{\tau}{\lambda^S}\frac{u'(\overbrace{1}^{c_1^B(0,0)})}{u'(\underbrace{\lambda^S}_{c_1^B(0,1)})} \tag{22}$$

If the cap θ satisfies condition (22) and also satisfies $\theta < 1 - \lambda^S$, then $\tilde{\alpha}_B^* = D_{SB,B}^* = 0$ is the unique type B equilibrium of the economy. The left hand side (LHS) of condition (22) is increasing in θ while the right hand side (RHS) does not change in θ. For $\theta = 0$ we have $LHS < RHS$ and for $\theta = 1 - \lambda^S$ we have $LHS > RHS$ (which follows from assumption 6.1). Hence there is a $\hat{\theta}_B$ with $0 < \hat{\theta}_B < 1 - \lambda^S$ so that condition (22) is fulfilled if and only if $\theta \geqslant \hat{\theta}_B$.

B Solving for the type A equilibrium

The relevant choice variable in a type A equilibrium is $\tilde{\alpha}_A$, the share of households' endowment invested into the SB sector. Denote $c_1^A(\tilde{\alpha}_A, \xi)$ as consumption of a household at t=1 if sunspot ξ realizes, given the economy is in a type A equilibrium. Denote $T(\xi)$ as the tax levied by the deposit

insurance agency, minus the fee revenue rebated. We have:

$$\overbrace{c_1^A(\tilde{\alpha}_A, 0)}^{\substack{\text{consumption}\\ \text{if no run}}} = \tilde{\alpha}_A \overbrace{1}^{\substack{\text{return}\\ \text{to SB}\\ \text{deposits}}} + (1-\tilde{\alpha}_A) \overbrace{(1-\tau)}^{\substack{\text{return}\\ \text{to CB}\\ \text{deposits}}} - T(0) \tag{23}$$

$$\overbrace{c_1^A(\tilde{\alpha}_A, 1)}^{\substack{\text{consumption}\\ \text{if run}}} = \tilde{\alpha}_A \overbrace{\frac{\lambda^S}{D(\mathcal{I}_{SB})}}^{\substack{\text{return}\\ \text{to SB}\\ \text{deposits}}} + (1-\tilde{\alpha}_A) \overbrace{(1-\tau)}^{\substack{\text{return}\\ \text{to CB}\\ \text{deposits}}} - T(1) \tag{24}$$

Households choose $\tilde{\alpha}_A$ such as to maximize expected utility, given by:

$$E[u(c_1^A(\tilde{\alpha}_A, \xi))] = \overbrace{(1-\pi^r)}^{\substack{\text{probability}\\ \text{of no run}}} u(c_1^A(\tilde{\alpha}_A, 0)) + \overbrace{\pi^r}^{\substack{\text{probability}\\ \text{of run}}} u(c_1^A(\tilde{\alpha}_A, 1)) \tag{25}$$

We have that:

$$\frac{d\,E[u(c_1^A(\tilde{\alpha}_A, \xi))]}{d\,\tilde{\alpha}_A} = (1-\pi^r)\,\tau\,u'(c_1^A(\tilde{\alpha}_A, 0)) + \pi^r \left[\frac{\lambda^S}{D(\mathcal{I}_{SB})} - (1-\tau)\right] u'(c_1^A(\tilde{\alpha}_A, 1)) \tag{26}$$

And:

$$\frac{d^2\,E[u(c_1^A(\tilde{\alpha}_A, \xi))]}{d\,\tilde{\alpha}_A^2} = \overbrace{(1-\pi^r)\,\tau^2}^{>0} \overbrace{u''(c_1^A(\tilde{\alpha}_A, 0))}^{<0} + \overbrace{\pi^r \left[\frac{\lambda^S}{D(\mathcal{I}_{SB})} - (1-\tau)\right]^2}^{>0} \overbrace{u''(c_1^A(\tilde{\alpha}_A, 1))}^{<0} < 0 \tag{27}$$

Since expected utility is continuous, and strictly concave in $\tilde{\alpha}_A$, the optimal choice of $\tilde{\alpha}_A$ is unique, as well as continuous in all the arguments. From the discussion in the main text we know that we only need to consider interior solutions. To solve for the equilibrium $\tilde{\alpha}_A^* = D_{SB,A}^*$ we insert the market clearing condition $D(\mathcal{I}_{SB}) = \tilde{\alpha}_A$ into expression (26), cancel out the deposit insurance fee payments, set expression (26) to zero and solve it for $\tilde{\alpha}_A$. This yields the following condition:

$$\left[(1-\tau) - \frac{\lambda^S}{\tilde{\alpha}_A}\right] \frac{u'(\overbrace{(1-\tilde{\alpha}_A) + \lambda^S}^{c_1^A(\tilde{\alpha}_A,1))})}{u'(\underbrace{1}_{c_1^A(\tilde{\alpha}_A,0)})} = \frac{1-\pi^r}{\pi^r}\,\tau \tag{28}$$

The left hand side (LHS) of expression (28) is continuous and strictly increasing in $\tilde{\alpha}_A$, while the right hand side (RHS) is a constant. For $\tilde{\alpha}_A = \lambda^S$ we have that $LHS < RHS$ and for $\tilde{\alpha}_A = 1$ we have that $LHS > RHS$ (which follows from assumption 6.1). This confirms that there is unique $\tilde{\alpha}_A = \tilde{\alpha}_A^* = D_{SB,A}^* \in (\lambda^S, 1)$ solving equation (28). Note also that, all else equal, an increase in π^r shifts RHS downwards, an increase in λ^S shifts LHS downwards, an increase in households' risk aversion shifts LHS upwards, and an increase in τ shifts RHS upwards and LHS downwards. This leads to the comparative static results mentioned in the main text.

C Solving for the type B equilibrium

In a type B equilibrium, the relevant choice variable of households is $\tilde{\alpha}_B$, the share of the uninsurable part of the endowment invested into the SB sector. Denote $c_1^B(\tilde{\alpha}_B, 0)$ and $c_1^B(\tilde{\alpha}_B, 1)$ as consumption in case of no run and a systemic run respectively, given the economy is in a type B equilibrium. Note that, in a type B equilibrium we have that $\vartheta_{CB} < 1 - \lambda^S$, since CBs would not be susceptible to runs otherwise. We have:

$$\begin{aligned}
\overbrace{c_1^B(\tilde{\alpha}_B, 0)}^{\substack{\text{consumption}\\ \text{if no run}}} &= \overbrace{\tilde{\alpha}_B(1-\theta)}^{\text{SB deposits}} + \overbrace{[\theta + (1-\tilde{\alpha}_B)(1-\theta)]\,(1-\tau)}^{\text{CB deposits}} - T(0) = (1-\tau) + \tilde{\alpha}_B(1-\theta)\,\tau - T(0) \\
\overbrace{c_1^B(\tilde{\alpha}_B, 1)}^{\substack{\text{consumption}\\ \text{if run}}} &= \overbrace{\tilde{\alpha}_B(1-\theta)\lambda^S}^{\text{SB deposits}} + \Big[\overbrace{(1-\tilde{\alpha}_B)(1-\theta)\frac{\lambda^S}{1-\vartheta_{CB}}}^{\text{uninsured CB deposits}} + \overbrace{\theta}^{\substack{\text{insured}\\ \text{CB deposits}}}\Big](1-\tau) - T(1) \\
&= \theta(1-\tau) + (1-\theta)\left[\tilde{\alpha}_B + (1-\tilde{\alpha}_B)\frac{1-\tau}{1-\vartheta_{CB}}\right]\lambda^S - T(1)
\end{aligned} \tag{29}$$

Households choose $\tilde{\alpha}_B$ such as to maximize expected utility, given by:

$$E[u(c_1^B(\alpha, \xi))] = (1-\pi^r)\,u(c_1^B(\alpha, 0)) + \pi^r u(c_1^B(\alpha, 1))$$

Derivation of expected utility with respect to $\tilde{\alpha}_B$ yields:

$$\frac{d\,E[u(c_1^B(\tilde{\alpha}_B,\xi))]}{d\,\tilde{\alpha}_B} = (1-\pi^r)\,(1-\theta)\,\tau\,u'(c_1^B(\tilde{\alpha}_B,0)) + \pi^r(1-\theta)\lambda^S\left(\frac{\tau-\vartheta_{CB}}{1-\vartheta_{CB}}\right)u'(c_1^B(\tilde{\alpha}_B,1)) \quad (30)$$

As in section B of the appendix it is straightforward to show that $\frac{d^2\,E[u(c_1^B(\tilde{\alpha}_B,\xi))]}{d\,\tilde{\alpha}_B^2} < 0$. Hence expected utility is continuous, as well as strictly concave in $\tilde{\alpha}_B$, so that the optimal choice $\tilde{\alpha}_B^{opt}$ is unique and continuous in all the arguments.

To solve for the range $[\hat{\theta}_B, 1-\lambda^S)$ for which the type B equilibrium is characterized by a corner solution with no SBs ($\tilde{\alpha}_B^* = 0$), we solve (22) for θ, which gives us the threshold $\hat{\theta}_B$. Note next that corner solutions with $\tilde{\alpha}_B = 1^*$ are not possible if $\theta > 0$. To see this, consider the following: If all uninsured deposits are held at SBs ($\tilde{\alpha}_B = 1$) then all deposits at CBs are insured ($\vartheta_{CB} = 1$). By (20) this means that households' optimal choice is to invest only in CBs $\tilde{\alpha}_B^{opt} = 0$. It follows that, whenever $\theta < \hat{\theta}_B$, the economy exhibits a type B equilibrium with an interior solution $\tilde{\alpha}_B^* \in (0,1)$ and $D_{SB,B}^* \in (0, 1-\theta)$. To solve for interior equilibria, we insert the market clearing condition $\vartheta_{CB} = \frac{\theta}{1-(1-\theta)\tilde{\alpha}_B}$ into expression (30), cancel out all transfers to- and from deposit insurance, set the expression to zero and solve for $\tilde{\alpha}_B$. If $\theta = 0$, then only SBs exist by definition.