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In Search of Optimal Liquidity for Deposit Insurers

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Abstract

Liquidity is a critical resource for deposit insurers. Liquidity can be obtained through liquid assets or borrowing capacity. The paper presents three approaches to setting liquidity targets. After discussing current methods, we argue that the two forms of liquidity can be viewed as stocks. Therefore, inventory optimization models can be applied. These models require that liquidity needs be identified by a distribution function, and that holding costs and stock-out costs be estimated. Using real data, the paper shows how such a model can be used.

Introduction

In troubled times, liquidity is a critical resource for deposit insurers. Sufficient liquidity allows a deposit insurer to reimburse depositors promptly, maintain confidence in the banking sector and thus contribute to stabilizing the financial system. Conversely, a lack of liquidity may prevent a deposit insurer from fulfilling its role. Deposit insurers are fully aware of this. They deploy great efforts to identify their liquidity needs and to gather liquidity as needed.

In finance, liquidity is generally defined as the ability to meet short-term obligations. For deposit insurers, short-term obligations arise when one or more insured banks become insolvent, forcing them to reimburse insured depositors. Thus, for most deposit insurers liquidity needs are infrequent but sudden and sizable, making the problem of forecasting liquidity needs very difficult.

Liquidity takes two main forms: it can be stored in liquid assets or obtained through borrowing capacity. These two forms are similar to the concepts of ex-ante and ex-post funding used by deposit insurers. Typically, liquid assets are considered the best form of liquidity because access to these assets is the most certain and immediate. However, this form is also seen as the most costly. Liquidity through liabilities is less certain and immediate. It is thus a somewhat inferior form of liquidity, but is usually much less costly. Generally, liquidity managers seek a hybrid solution using both forms of liquidity resources and trading off their pros and cons.

The purpose of this paper is to describe various possible approaches to identifying optimal liquidity for deposit insurers. Three approaches will be discussed, ranging from the most qualitative to the most quantitative.

First, the paper presents the most popular approach, which links liquidity to ex-ante funding. Second, it shows how a value-at-risk approach can be used when liquidity demand can be characterized by a distribution function. Finally, it argues that liquidity can be viewed as a stock, and shows how inventory theory can be used to identify its optimal level. A numerical example using real data will be provided for illustration.

1. Liquidity target and ex-ante funding

Traditionally, deposit insurers have focused on obtaining sufficient ex-ante funding through premiums collected from insured banks. Setting a target for ex-ante funding is typically expressed as a percentage of insured deposits. From a liquidity management perspective, this natural approach can be interpreted as follows. In case of bank insolvency, liquidity needs would be proportional to insured deposits. Insured deposits are thus used as a proxy for liquidity risk or liquidity needs. As such, premiums allow deposit insurers to build their capital. In turn, this capital is invested in liquid assets. Thus, this approach tries to balance liquidity reserves with liquidity needs.

However, identifying the appropriate percentage of insured deposits as a target for ex-ante funding is a significant challenge because many factors may affect liquidity needs, such as the probabilities of insolvency of insured banks, the nature of their assets, the composition of their liabilities and the systemic factors that may trigger multiple bank failures. Given that these factors differ between countries, it is not surprising that targets for ex-ante funding vary considerably across deposit insurers.

The ability of insured banks to contribute to ex-ante funding is often limited, and constrains deposit insurers to settle for a pragmatic target somewhat below the level required to meet liquidity needs. This naturally leads deposit insurers to complement liquid assets by borrowing capacity, and wisely so. This two-pronged approach grants deposit insurers access to a larger pool of liquidity without putting too much strain on insured banks.

To summarize, this approach is intuitive but requires considerable judgment on the part of deposit insurers to qualitatively assess many complex factors affecting potential liquidity needs and the ability of insured banks to finance liquid assets through premiums. The result is often a pragmatic and somewhat political trade-off between liquidity supply and potential liquidity demand.

2. Liquidity target through a value-at-risk approach

Under this approach, a deposit insurer would try to identify the distribution of liquidity needs, most likely using a simulation model based on historical data and assumptions about the future. Several deposit insurers currently run simulation models to evaluate the distribution of their losses. Such models could be useful to simulate liquidity needs, but would require some adaptation arising from the difference between the notions of loss and liquidity need. A loss is the difference between revenues and costs. In the case of bank liquidation, reimbursements to insured depositors represent a cost to the insurer that is reduced by the revenues obtained from the eventual sale of the usually illiquid assets seized from the insolvent bank. Thus, liquidity needs generated by immediate compensation of depositors are generally greater than the estimated losses, which take into account future revenues from the sale of seized assets.

Assuming that a distribution of liquidity needs has been identified through a simulation model, it would be possible to characterize the target level of liquidity using a value-at-risk or liquidity-at-risk approach. A level of confidence is specified that states the desired probability of having enough liquidity. Bearing in mind that liquidity can take one of two forms— liquid assets and liquidity through liabilities—, a liquidity policy could be expressed as follows: "Given a distribution of liquidity needs, liquid assets should be adequate with a probability of 60% and total liquidity (liquid assets plus borrowing capacity) should be adequate with a probability of 90%." Provided with a given distribution, one could easily identify the two target levels of liquidity in the unit of currency.

This approach is superior to the approach described in section 1 because it tries to model the distribution of liquidity demand explicitly. However, the final step of stating the probability levels for the adequacy of liquidity supply remains judgmental and leaves room for arbitrariness. The purpose of the next section will be to present an explicit and rational approach to setting these probabilities.

3. Identifying optimal liquidity using inventory theory

3.1 Identifying the optimal level of stock with inventory theory

This approach views liquidity as a stock awaiting uncertain demand. Inventory theory can be used to identify the optimal level of stock. To solve the problem, one needs to characterize the uncertain demand via a distribution function F(x), where F(x) is the probability that demand will be less than or equal to x. One then needs to assess the value of two types of costs associated with inventory theory, namely: holding costs (Ch) and stock-out costs (Cs). Typically, for a retailer, holding costs include the cost of financing and storing inventory, and stock-out cost is the opportunity cost of missing a sale and losing the profit on that sale.

There are several ways to solve this optimal inventory problem. The most intuitive method is based on the principle that at the optimum, expected marginal holding costs will be equal to expected marginal stock-out costs. These expected marginal costs can be expressed as follows:

Expected marginal holding costs:	$E(Ch) = F(x)\ Ch$
Expected marginal stock-out costs:	$E(Cs) = (1-F(x))\ Cs$
Optimality condition:	$E(Ch) = E(Cs)$
Which develops as:	$F(x)\ Ch = (1-F(x))\ Cs$
From this, we get:	$F(x) = Cs / (Ch+Cs)$

This last equation identifies the optimal probability or level of safety given holding costs and stock-out costs. For example, a retailer with a stock-out cost of 100% and a holding cost of 25% should hold stocks to satisfy demand (1/(.25+1))= 80% of the time.

To find the optimal stock level, one need only invert the distribution function. This is expressed as:

$$x = F^{-1}(Cs / (Ch+Cs))$$

The characterization of optimal stocks is thus fairly simple using inventory theory.[1] The challenge is now to apply this result in the context of deposit insurance.

3.2 Applying the optimal stock result to identify optimal liquidity for a deposit insurer

Assuming that the distribution of liquidity needs has been identified, as discussed in section 2, and taking into account that liquidity comes in two forms, which may imply different costs, one would then need to evaluate four parameters in order to use the optimality result, namely holding costs and stock-out costs for liquid assets and holding costs and stock-out costs for borrowing capacity. This is by no means a simple task, but one worth tackling. I will audaciously make some suggestions in this regard, hoping at least to provide food for thought and discussion.

I suggest that holding costs for the liquid assets of a deposit insurer are an opportunity cost equal to the difference between the yield that could be obtained by the insured bank on the premium funding the liquid assets of the deposit insurer and the yield effectively obtained by the deposit insurer on its liquid assets. For example, assuming that under competitive markets premium are paid out of the capital of the insured bank and that it earns a 15% return on equity, and assuming that the deposit insurer conservatively invests its liquidity in safe assets providing a yield of 3%, then it could be argued that the holding cost is the forgone yield of 12%. Although this evaluation is debatable, evaluating stock-out costs is even more challenging.

I believe that running out of liquid assets basically means resorting to using borrowing capacity. Normally, this operation involves transaction or issuing costs, which can be implicit, such as internal administrative costs, or explicit, such as fees to issue debt. Also, one would need to include any interest to be paid on the borrowed money. Only the treasurer of a deposit insurer can attempt to evaluate such costs.

As for the liquidity obtained through borrowing capacity, I contend that the holding cost of such liquidity is not zero. Theoretically, if the cost was zero, all deposit insurers would have infinite borrowing capacity, and clearly this is not the case, which suggests that

[1] In inventory theory, this optimization problem is known as the "Newsvendor problem." Khouja (1999), Qin et al (2011), Choi (2012) and Chen et al. (2016) present the rich literature on this problem and its many variants.

there is an implicit cost. I will analyze this situation by analogy. When a corporate borrower applies for a line of credit at his bank, the bank computes the required capital to assume the credit and liquidity risks, multiplies this amount of capital by its required rate of return, and thus gets the revenue it requires on the unused part of the line of credit and the rate to be charged. For example, if a bank requires 4% of capital on the unused portion of a line of credit and targets a 15% rate of return on capital, it should charge 60 basis points on the unused part of the line of credit, i.e. the borrowing capacity offered to its corporate client. I would argue that this analogy applies to a deposit insurer, although the two parameters could be somewhat lower. Alternatively, borrowing capacity may be viewed as an option to borrow, and in finance an option has value and consequently a price.

Finally, one would need to evaluate the stock-out costs relative to the borrowing capacity. This is perhaps the most difficult parameter to estimate. One would need to envision what would happen if borrowing capacity were insufficient. Would there be still some emergency funding available, and if so, at what cost? Might the deposit insurer be unable to honor its obligations? In such a case, what would the costs in terms of lost confidence from depositors and potential increased instability of the banking sector be? Only practitioners in a real context can tackle this analysis.

Nonetheless, to illustrate the mechanics of the inventory model, the next section will provide a numerical example inspired by public data from the Canada Deposit Insurance Corporation (CDIC).

3.3 A numerical example

The following numerical example is based on public data taken from the 2017 annual report of the Canada Deposit Insurance Corporation (CDIC). The table below, "Ex ante funding," shows that total capital, consisting of retained earnings and provision for insurance losses, is equal to $3.836 B.

Ex ante funding

	Actual		Target
As at March 31 (C$ thousands)	**2017**	**2016**	**2017**
Retained earnings	2,235,979	2,116,266	
Provision for insurance losses	1,600,000	1,300,000	
Total *ex ante* funding	**3,835,979**	**3,416,266**	**7,413,282**
Total basis points of insured deposits	**52***	**49****	**100**

* Based on level of insured deposits as at April 30, 2016. Includes changes in insured deposit levels as a result of changes to the membership during the year ended March 31, 2017.

** Based on level of insured deposits as at April 30, 2015.

Next, the table “Available Funds” on page 26 shows that liquid assets, consisting of cash and liquid investment securities, are equal to \$3.837 B.

As at March 31 (C\$ millions)	2017	2016
Available liquid funds:		
Cash	2	1
Fair value of high quality, liquid investment securities	3,835	3,449
Availability of borrowings:		
Borrowings authorized under the *CDIC Act*, either from market sources or from the Consolidated Revenue Fund	22,000	20,000
Total available funds	**25,837**	**23,450**

Comparing the value of capital (\$3.836 B) and liquid assets (\$3.837 B) supports the view that the two values are essentially equal, and that capital is invested mainly in liquid assets. Thus, we will consider that the 1% of insured deposits as a target for ex ante funding is de facto a 1% target for liquid assets.

Appendix 1 illustrates the application of the inventory optimization model to the situation of the CDIC. Section 1 summarizes the data from the two tables above. Section 2 presents our assumptions. Section 2.1 states our hypotheses about holding costs and stock-out costs. We assume that the holding cost is 10% for liquid assets, estimated roughly as the difference between the return on equity of insured banks and the return on investment obtained by CDIC. The holding cost is assumed to be 30 basis points for borrowing capacity, estimated as somewhat lower than the cost of a line of credit for a private corporation. Stock-out costs are assumed to be 15% in both cases. Computing the optimal level of safety (OLS) based on these costs, liquid assets should be sufficient with a 60% probability, while the borrowing capacity should be sufficient with a probability of 98.04%. To go from these optimal levels of safety to dollar amounts, one needs to know the distribution of liquidity needs. In Section 2.2, we assume that this distribution is an exponential distribution that has only one parameter, λ. This distribution is consistent with the assumption that the probability of a liquidity need decreases exponentially with the size of the need. The exponential distribution and its inverse function are expressed as below:

$$F(x) = 1 – e^{-\lambda x}$$

$$F^{-1}(x) = \ln(1-x) / -\lambda$$

Appendix 2 displays the probability distribution and the cumulative probability distribution for this exponential function. The parameter λ was set at a value of 0.12365 using the assumption that the 1% target for liquid assets would indeed place CDIC at the optimum, i.e. at an optimal level of safety of 60% based on the assumed holding and stock-out costs.

First, note that the procedure for identifying the optimal level of a stock has been derived for the case of an inventory of one item. Clearly, this procedure can be applied in isolation for the two components of liquidity, i.e. liquid assets and borrowing capacity, as if they were independent. However, in reality these two stocks are not independent: they can substitute for one another. Formally, this inventory problem is identified as a multi-product problem. Several authors have studied this problem, such as Khouja et al. (1996), Turken et al. (2012) and Deflem and Van Nieuwenhuyse (2013). In general, two factors may create dependence between the items: a budget constraint and the possibility of substituting one item for another. These two types of interactions may apply in the case of liquid assets and borrowing capacity. As the articles above show, the formal solution to these problems is possible but tends to be complex. Because I have imperfect information on the distribution function and on the various costs, I do not believe that using the formal multi-product approach at this point is justified. Rather I propose to use an informal or heuristic approach to estimate the lower and upper bounds of total liquidity depending on the assumptions about the relationship between liquid assets and borrowing capacity. The first case would be that there is no relationship (i.e. the optimal level of each variable is independent of the other), in which case total liquidity would be simply the sum of these two components. This case generates an upper bound on optimal total liquidity. The second case would be that liquid assets are a perfect substitute for borrowing capacity, in which case one would subtract the optimal amount of liquid assets from the optimal borrowing capacity identified under independence to obtain the optimal amount of borrowing capacity under the substitution hypothesis. This case generates a lower bound on optimal total liquidity.

Section 3 of Appendix 1 shows the results of our analysis for three scenarios. The first scenario is based on the actual values of liquid assets and borrowing capacity. The table shows that these values would provide levels of safety of 37.78% and 95.90% respectively, compared with the optimal levels of 60% and 98.04%. The second scenario is based on the assumption that the 1% target for liquid assets is achieved and is optimal. In this scenario, borrowing capacity remains at its current value. Liquid assets would provide the optimal level of safety of 60%, while total liquidity would reach a level of safety of 97.37%, slightly below the optimal value of 98.04%. Finally, the third scenario shows the level of liquid assets and borrowing capacity needed to achieve the optimal level of safety of 98.04%. The value of the optimal capacity varies whether the two forms of liquidity are independent or if substitution applies. In the case of independence, total liquidity would be the sum of the two forms of liquidity and would provide a level of safety of 99.22%. Under the assumption that liquid assets partly substitute for borrowing capacity, total liquidity would provide a level of safety of

98.04%. The lower bound of borrowing capacity would thus be $31.80 B and the upper bound would be $39.21 B. Clearly, all these results are dependent on our assumptions about the costs and the distribution of liquidity needs.

A typical approach to deal with uncertainty about assumptions is to perform sensitivity analyses. Appendix 3 shows a sensitivity analysis for holding costs and stock-out costs for liquid assets. The graph displays optimal liquid assets as a function of these two costs. Similarly, Appendix 4 presents a sensitivity analysis and its graph for the optimal borrowing capacity. In both cases, the optimal value of the liquidity component decreases with higher holding costs and increases with higher stock-out costs, which is consistent with intuition.

The main purpose of this example was to show how inventory theory can be applied in the context of deposit insurance. To summarize, it is necessary to first estimate the distribution of liquidity needs and the holding and stock-out costs. The inventory model then allows us to identify optimal levels of safety and optimal amounts of liquid assets and borrowing capacity. The final solution for the aggregate amount of liquidity resources depends on the assumption made about the relationship between the two components of liquidity.

Conclusion

Like all insurers, deposit insurers face uncertain demand for compensation. To assume this risk responsibly, deposit insurers must build sufficient capital and in turn make sure to have adequate liquidity when need arises. Liquidity is a critical resource to maintain confidence in the banking system in troubled times and thus assure its stability. Liquidity is obtained not only through liquid assets but also through borrowing capacity.

Traditionally, deposit insurers measure risk by the amount of insured deposits, and have thus stated their target capital as a percentage of insured deposits. However, this measure is admittedly crude because it does not consider the risks originating from the assets of insured banks. Consequently, deposit insurers have turned to simulation models to evaluate their risks better. Although this new approach marks definite progress, it does not provide a methodology to identify the optimal amount of capital or liquidity. These decisions remain very judgmental.

The main purpose of this paper was to argue that liquidity can be viewed as a stock, and that inventory theory can be used to identify its optimal level. Given the characterization of uncertain demand for liquidity by a distribution function, only two more pieces of information are needed: holding costs and stock-out costs. Similar to the optimization of any stock, the principle leading to the solution is to balance these two opposing costs. A numerical example based on public data showed that this theoretical approach can indeed be put into practice.

This paper could convince managers at deposit insurers that estimating both the holding and stock-out costs associated with their liquidity resources is an interesting and promising next step in their search for optimal liquidity.

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Appendix 1
Optimization model for liquidity management at a deposit insurer

1. Data from the Canada Deposit Insurance Corporation 2017

Variables	Symbol	Actual Values	Actual Ratios	Target Values	Target Ratios
Liquid assets	LA	3.837	0.52%	7.41	1.00%
Borrowing capacity	BC	22.000	2.97%	22.00	2.97%
Total liquidity	TL	25.837	3.49%	29.41	3.97%
Insured deposits	ID	741			

2. Hypotheses on marginal costs and distribution of liquidity demand

2.1 Optimal Level of Safety (OLS) as a function of marginal costs

Marginal costs		Holding	Stock-out	OLS
		Ch	Cs	Cs/(Ch+Cs)
Liquid assets	LA	0.10	0.15	0.600
Borrowing capacity	BC	0.0030	0.15	0.9804

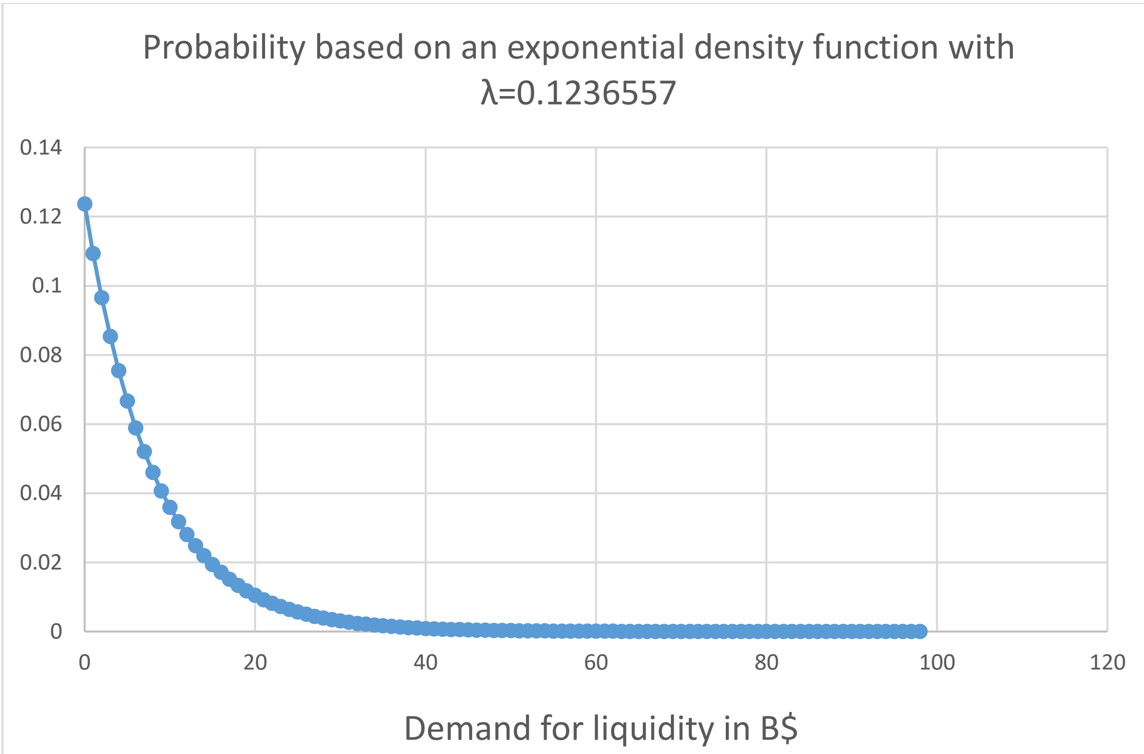
2.2 Assumption regarding the distribution of liquidity demand

Exponential distribution		
Density function	f(x)	$\lambda e^{-\lambda x}$
Cumulative distribution	F(x)	$1-e^{-\lambda x}$
Lambda	λ	0.1236557
Expected value	E	8.09
Standard deviation	σ	8.09
Median value	m	5.61

3. Results of the model for three scenarios

		Value	Probability	Ratio
		V	F(V)	V/ID
Actual situation				
Liquid assets	LA	3.84	37.78%	0.52%
Borrowing capacity	BC	22.00	93.42%	2.97%
Total liquidity	TL=LA+BC	25.84	95.90%	
Targeted situation				
Liquid assets	LA	7.41	60.00%	1.00%
Borrowing capacity	BC	22.00	93.42%	2.97%
Total liquidity	TL=LA+BC	29.41	97.37%	3.97%
Optimal situation				
Liquid assets	LA	7.41	60.00%	1.00%
Borrowing capacity	BC	31.80	98.04%	4.29%
Bor. cap. after substitution	BCAS	24.39	95.10%	3.29%
Total liquidity				
- under independence	TL=LA+BC	39.21	99.22%	5.29%
- under substitution	TL=LA+BCAS	31.80	98.04%	4.29%

Appendix 2


Probability based on an exponential density function with λ=0.1236557
0.14
0.12
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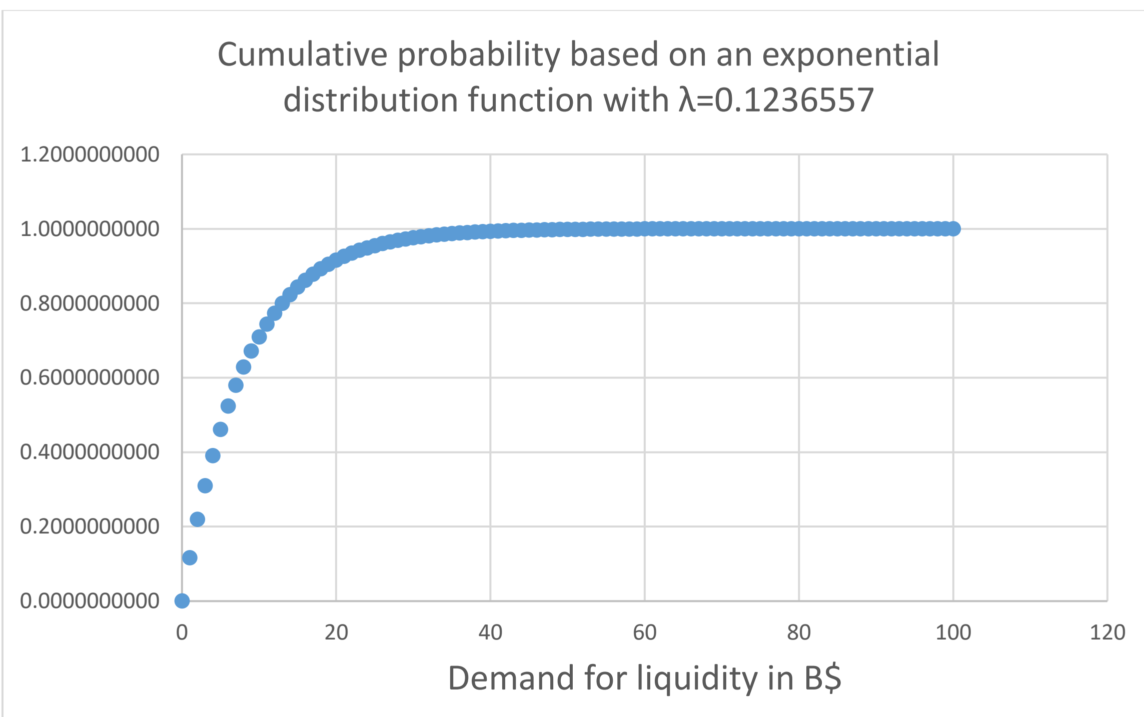
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100
120
Demand for liquidity in B$


Cumulative probability based on an exponential distribution function with λ=0.1236557
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1.0000000000
0.8000000000
0.6000000000
0.4000000000
0.2000000000
0.0000000000
0
20
40
60
80
100
120
Demand for liquidity in B$

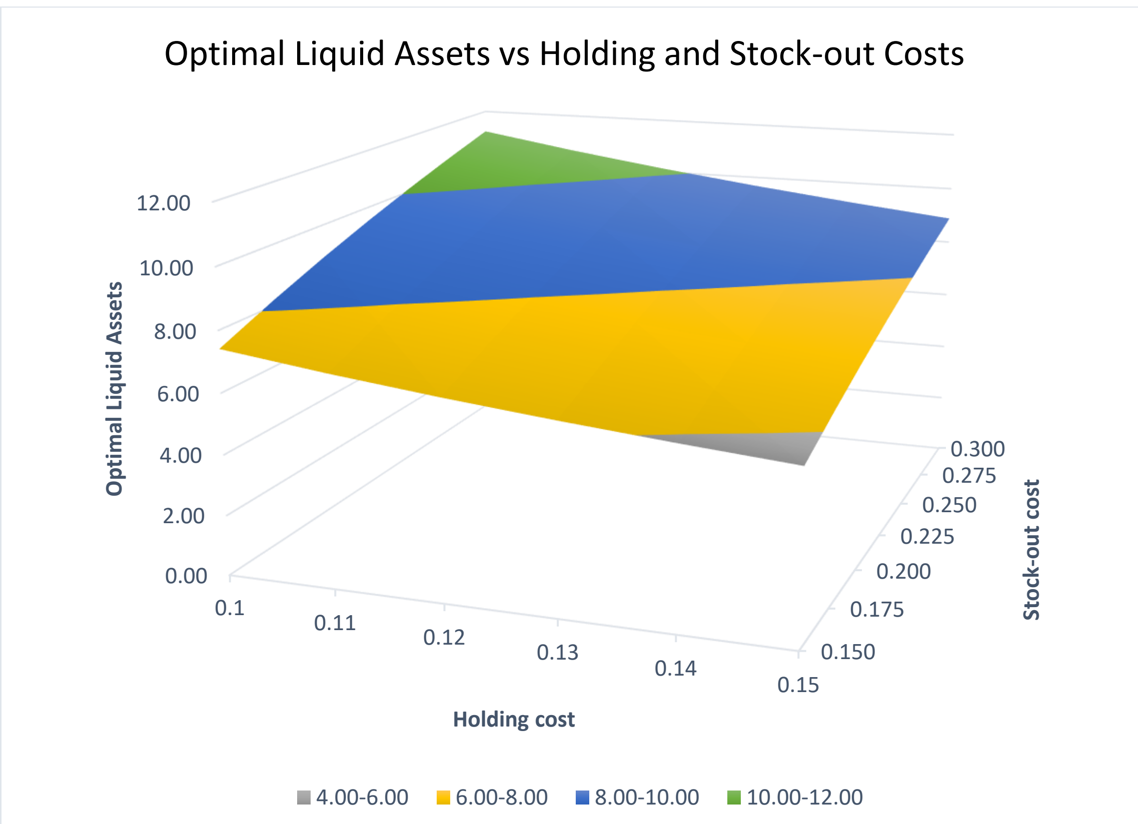
Appendix 3
Sensitivity analysis for liquid assets

Optimal levels of safety vs. holding cost (Ch) and stock-out cost (Cs)

Ch / Cs	0.150	0.175	0.200	0.225	0.250	0.275	0.300
0.1	60.0%	63.6%	66.7%	69.2%	71.4%	73.3%	75.0%
0.11	57.7%	61.4%	64.5%	67.2%	69.4%	71.4%	73.2%
0.12	55.6%	59.3%	62.5%	65.2%	67.6%	69.6%	71.4%
0.13	53.6%	57.4%	60.6%	63.4%	65.8%	67.9%	69.8%
0.14	51.7%	55.6%	58.8%	61.6%	64.1%	66.3%	68.2%
0.15	50.0%	53.8%	57.1%	60.0%	62.5%	64.7%	66.7%

Optimal liquid assets vs. holding cost (Ch) and stock-out cost (Cs)

Ch / Cs	0.150	0.175	0.200	0.225	0.250	0.275	0.300
0.1	7.41	8.18	8.88	9.53	10.13	10.69	11.21
0.11	6.96	7.70	8.38	9.01	9.59	10.13	10.64
0.12	6.56	7.27	7.93	8.54	9.11	9.63	10.13
0.13	6.20	6.90	7.53	8.12	8.67	9.19	9.67
0.14	5.89	6.56	7.18	7.75	8.29	8.79	9.26
0.15	5.61	6.25	6.85	7.41	7.93	8.42	8.88



Appendix 4
Sensitivity analysis for borrowing capacity

Optimal levels of safety vs. holding cost (Ch) and stock-out cost (Cs)

Ch / Cs	0.150	0.175	0.200	0.225	0.250	0.275	0.300
0.0015	99.0%	99.2%	99.3%	99.3%	99.4%	99.5%	99.5%
0.0020	98.7%	98.9%	99.0%	99.1%	99.2%	99.3%	99.3%
0.0025	98.4%	98.6%	98.8%	98.9%	99.0%	99.1%	99.2%
0.0030	98.0%	98.3%	98.5%	98.7%	98.8%	98.9%	99.0%
0.0035	97.7%	98.0%	98.3%	98.5%	98.6%	98.7%	98.8%
0.0040	97.4%	97.8%	98.0%	98.3%	98.4%	98.6%	98.7%

Optimal borrowing capacity vs. holding cost (Ch) and stock-out cost (Cs)

Ch / Cs	0.150	0.175	0.200	0.225	0.250	0.275	0.300
0.0015	37.32	38.56	39.63	40.57	41.42	42.19	42.89
0.0020	35.02	36.25	37.32	38.27	39.11	39.88	40.57
0.0025	33.24	34.47	35.54	36.48	37.32	38.09	38.78
0.0030	31.80	33.02	34.08	35.02	35.86	36.63	37.32
0.0035	30.58	31.80	32.86	33.79	34.63	35.39	36.09
0.0040	29.52	30.74	31.80	32.73	33.57	34.33	35.02